
Commodity Market Dynamics and Price Volatility: Insights from Dynamic Storage Models ¹

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ABSTRACT

The recent fluctuations of agricultural commodity prices have stimulated the debate on the potential causes of price volatility. The most common explanation is that weather shocks or other external factors perturb supply, thus leading to substantial price fluctuations. In view of the development of global markets, which tend to average out supply disturbances, one would expect price volatility to decrease if primarily caused by external shocks. This however is contradicted by the experience of the recent past. An alternative explanation proposes that the persistent fluctuations are the result of nonlinear dynamics and would even occur in the absence of external shocks. The focus of this paper is on the latter type of explanation. It is investigated under which conditions price volatility is primarily caused by nonlinear dynamics. A system dynamics modelling approach is used for the analysis. The model results show that plausible behaviour of actors can lead to persistent price fluctuations, even in the absence of external shocks.

Keywords: commodity cycles; nonlinear dynamics; price volatility; system dynamics

1 Introduction

It is well established that high price fluctuations of major food commodities have negative effects on welfare. The sharp increase of agricultural commodity prices in the recent past has therefore raised international concerns. The main questions in this context are: what drives the seemingly growing price fluctuations and how could they be reduced?

With regard to the causes of price volatility two kinds of explanations have been proposed. The most common one is *exogenous*, i.e. weather shocks or other exogenous factors perturb supply. In connection with low price elasticity of demand this leads to substantial price fluctuations. The other explanation is *endogenous* in that it proposes inherent fluctuations as result of nonlinear dynamics caused by factors like erroneous expectations, capacity constraints and time lags. Such fluctuations even occur in the absence of exogenous shocks. What primarily differentiates these two explanations is the type of expectations. While the endogenous explanation rests on backward-looking (naive or adaptive) expectations, the exogenous explanation is based on the concept of rational expectations (Gouel 2012, p138). In both theories, storage is of critical importance.

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The ‘exogenous fluctuation’ theory leads to the *competitive storage model*, as originally described by Gustafson (1958) and later on discussed frequently in the literature (e.g Muth 1961, Wright and Williams 1982, Deaton and Laroque 1992, 1995, 1996, Carter et al. 2011, Wright 2011, Cafiero et al. 2011, Guerra et al. 2015). In the theory of competitive storage, the price dynamics are determined by optimal reactions of agents (farmers, inventory holders) who use all available information to generate *rational expectations*, and derive optimal decisions with respect to purchasing and selling from those. Under these behavioural assumptions, storage has a stabilising impact and persistent price fluctuations are only caused by repeated random shocks.

The rational expectation hypothesis is controversially discussed. Studies which try to identify how agents form their expectations come to contradictory conclusions (cf. Guel 2012). While for example Miranda and Glauber (1993) find evidence that agents might follow rational expectations, Irwin and Thraen (1994) doubt that these results are robust. In a survey of several studies they rather recognise a lack of consensus regarding the formation of expectations. For the same market, one study supports the rational expectations hypothesis, while another one opts for adaptive or even for naive expectations. Thus, while the rational expectation hypothesis is a common assumption in agricultural economics, it cannot be taken for granted.

The endogenous explanation rests on the *cobweb* logic, originally popularised by Ezekiel (1938). The original (linear) cobweb model with its simple price trajectories is certainly far from reality, but it has been refined substantially over the past decades. While keeping the basic ingredients of the original model, i.e. imperfect expectations and time lags, the introduction of various forms of nonlinearities provide for complex dynamic behaviour, including strange attractors like limit cycles and chaotic motion. For example, Day (1982) obtains a chaotic trajectory from a generalised cobweb model. Mackey (1989) investigates the impacts of price dependent delays, and derives the conditions under which bifurcation leads into chaos. Chavas and Holt (1993) demonstrate that a nonlinear model of the US dairy industry could produce aperiodic price dynamics endogenously. Mitra and Boussard (2012) create an agricultural commodity model with storage that can produce a variety of price dynamics. Berg and Huffaker (2015) develop a nonlinear model of the German pork industry and apply it to reveal causal factors of the German hog-price cycle.

In reality we likely have both, exogenous shocks as well as nonlinear dynamics. In view of the development of global markets which tend to average out supply disturbances, one would expect price volatility to decrease, if primarily caused by external shocks. This however, is contradicted by the experience of the recent past. We therefore focus on the second type of explanation and investigate under which conditions price volatility is primarily caused by nonlinear dynamics. In the paper we will emphasize the nature of storage and its impacts on commodity price volatility. The analysis will be based on a *system dynamics* model aimed at identifying structures and parameters that can lead to persistent endogenous fluctuations. The model results shall reveal important factors which affect the dynamic properties of the system and determine the magnitude of price fluctuations.

2 Modelling approach

We consider an agricultural commodity that can be characterized as storable staple good. In this context, an important aspect is the role of storage and stock management. The *system dynamics* model therefore covers the production of a crop, its storage on farms and by distributors and finally its supply to processors or retailers. The structure of the model is illustrated by the stock and flow diagram of Figure 1. It is composed of three essential *feedback loops* capturing (A) the production process and material flow, (B) short term inventory management and (C) commodity price adjustment.

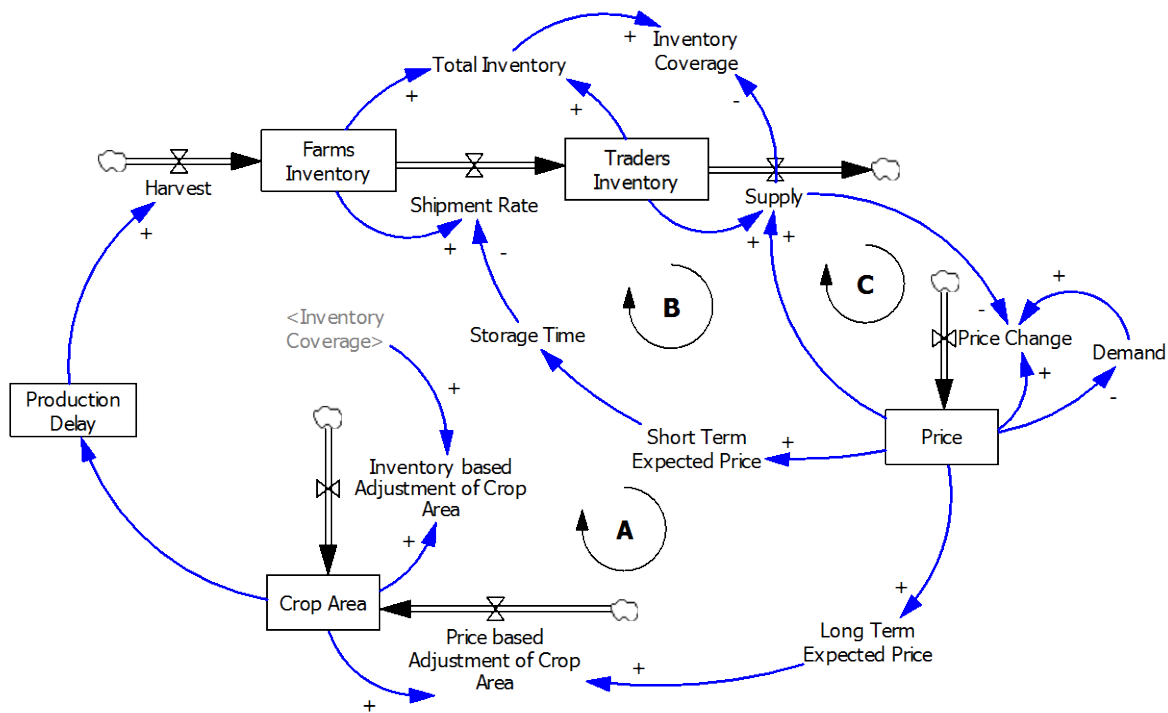


Figure 1: Stock and flow diagram of the system dynamics model

The behavioural hypotheses governing these feedback loops are the following:

- (A) *Production* is governed by the adjustment of the crop area, which is composed of two parts: price based adjustment and inventory based adjustment. The former depends on the long term price expectations such that rising prices cause an expansion of the crop area and vice versa. The inventory based adjustment of the crop area implements the assumption that low inventory levels relative to supply induce a production increase in order to prevent running out of stock in the future.
- (B) *Inventory management* is based on actual prices and short term price expectations influencing the commodity traders' purchasing and selling activities. Sales are captured by the variable 'supply' and depend on the current market price. In modelling the transfer from the on farm to the traders' inventory, Mackey's price dependent delay approach (Mackey 1989) is employed, where the variable 'storage time' represents the delay time.
- (C) The *adjustment of the commodity price* is modelled as a dynamic process. In this process the relative rate of change of the market price depends on the difference between demand and supply.

In summary these hypotheses constitute a nonlinear dynamic system the mathematical details of which are explained in the following paragraphs.

2.1 Price dynamics and demand

In considering price dynamics, assume that relative rate of change of the market price P depends on the difference between demand (D) and supply (S) as e.g. proposed by Mackey (1989, p 498). The process can then be described by the nonlinear differential equation:

$$\frac{1}{P} \frac{dP}{dt} = a (D(t) - S(t))$$

or

$$\frac{dP}{dt} = a (D(t) - S(t)) P(t) \quad , \quad a > 0$$

The following diagram (Figure 2) illustrates the price dynamics graphically. The process is composed of three feedback loops. A high price has a negative impact on demand while high demand affects price change positively. This constitutes in total a negative feedback loop which has a stabilizing effect. The same is true for the leftmost loop as high prices lead to high supply which, in turn, suppresses the subsequent price change. The feedback loop in the middle of the graph is a positive one and therefore amplifies the price change.

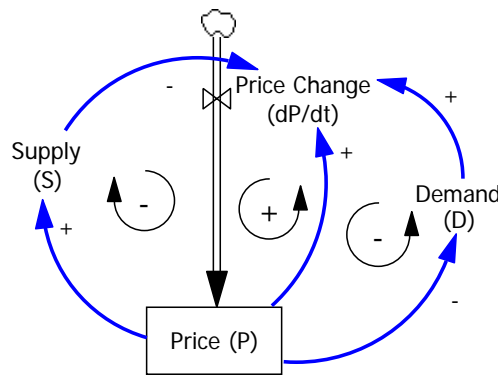


Figure 2: Diagram of price dynamics

Demand is modelled using an isoelastic demand function

$$D(t) = b P(t)^{-c} \quad , \quad c > 0$$

where c represents the price elasticity of demand and b is a scale factor. The reason for using an isoelastic demand function is primarily a technical one: we want to avoid impacts that varying elasticities would have on the model results.

2.2 Formation of price expectations

Price expectations are modelled as adaptive learning process. This conforms to the hypothesis of bounded rationality, assuming that only the information embodied in past the prices is used for predictions. A familiar way of modelling adaptive expectations is via *exponential smoothing*. In continuous time systems this model can be stated using the following differential equation, where $P^e(t)$ and $P(t)$ denote the expected and current price, respectively, and τ symbolizes the adjustment time lag:

$$\frac{dP^e}{dt} = \frac{1}{\tau} (P(t) - P^e(t)) \quad , \quad \tau > 0$$

This formula is used for long term as well as short term price expectations, where the time lag τ is set to 2 years for the long term and to 0.5 years for the short term expectations.

2.3 Storage and supply

The stock and flow diagram of Figure 3 delineates the model of the supply chain, which is composed of two storage compartments: the on farm inventory (*FI*) and the inventory held by distributors (*TI*). The harvest enters the on farm stock from where it is shipped to the distributors. From there it is finally supplied to processors or retailers.

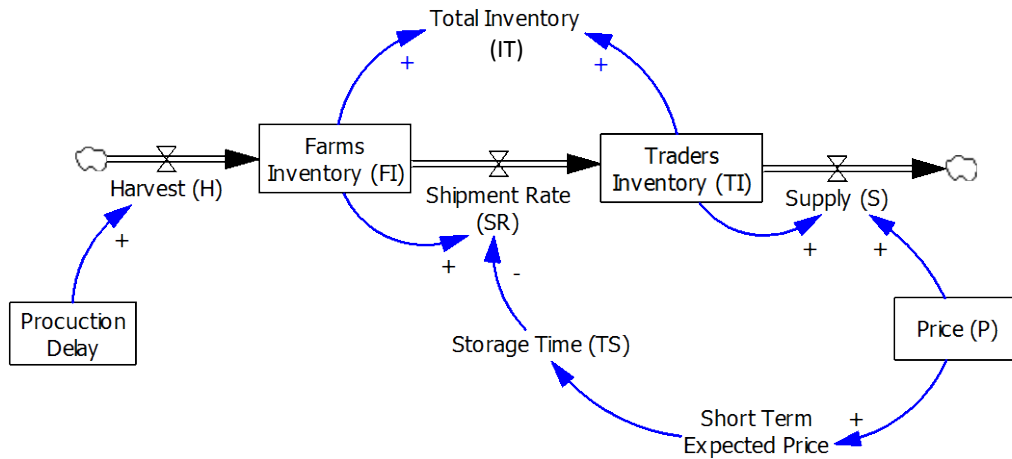


Figure 3: Supply chain of the model

Each compartment is represented by a differential equation, i.e.

$$\frac{dFI}{dt} = H(t) - SR(t)$$

and

$$\frac{dTI}{dt} = SR(t) - S(t)$$
(4)

The shipment rate from farmers to traders (*SR*) is determined by the average storage time (*TS*), i.e. $SR(t) = FI(t)/TS(t)$. The on farm storage time is a variable depending on the short term price expectation. Typically, one would assume that increasing price expectations lead to a shorter storage period with the maximum occurring in the neighbourhood of a perceived equilibrium price, i.e. a price covering the production cost. The reason for this is that high expected prices for the near future would intensify the traders' efforts to mobilise on farm stocks. If the expected price falls much below this cost price the storage period again is likely to fall as producers attempt to minimize their losses. This relationship is delineated in Figure 4.

Mathematically the curve of Figure 4 can be represented using the following function to compute *TS*:

$$TS(P^{es}) = TS^* \frac{P^{es}(t)}{P^*} e^{\left(1 - \frac{P^{es}(t)}{P^*}\right)}, \quad TS^*, P^* > 0$$
(5)

In the above formula TS^* denotes the maximum storage period, P^* is the perceived cost price and P^{es} represents the short term expected price. From this the shipment rate *SR* becomes:

$$SR(t) = \frac{FI(t)}{TS^* \frac{P^{es}(t)}{P^*} e^{\left(1 - \frac{P^{es}(t)}{P^*}\right)}} \quad (6)$$

Finally, at a given point in time the supply is a function of the (marginal) storage cost and the available stock at hand. Assuming a linear marginal cost function the supply S is given by

$$S(t) = TI(t) g P(t), \quad g > 0 \quad (7)$$

where g represents the unit marginal cost and TI denotes the currently available inventory. Thus, the actual inventory limits the instantaneous supply.

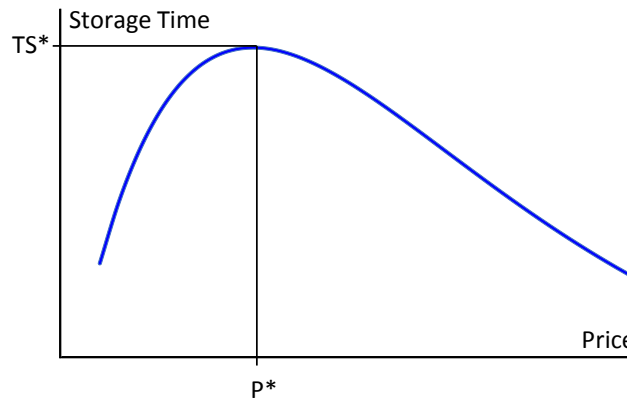


Figure 4: Storage time as a function of price

2.4 Dynamics of production

Restoring the commodity stock is a dynamic process which involves (1) adjusting the crop area and (2) a time lag which accounts for the time necessary to produce the crop, i.e. from one harvest to the next. Adjusting the crop area CA follows a dynamic process which is modelled by the differential equation

$$\frac{dCA}{dt} = PA(t) + IA(t) \quad (8)$$

where the variable $PA(t)$ denotes the *price based* adjustment of the crop area at time t , and $IA(t)$ represents the respective *inventory based* adjustment. The former depends on the long term price expectation such that $PA(t)$ becomes:

$$PA(t) = w(v P^{el}(t) - CA(t)), \quad w, v > 0 \quad (9)$$

In the above formula $P^{el}(t)$ denotes the expected market price, so $v P^{el}(t)$ marks the upper limit of the process, which can be interpreted as the “target crop area”, which is assumed to be proportional to the expected market price. A falling market price can cause a reduction of the crop land if the term inside the brackets becomes negative as the current crop area $CA(t)$ exceeds $P^{el}(t)$. Rising market prices, in turn, would lead to an extension of crop land. The parameter w indicates the speed of the adjustment process.

The second term in the differential equation (8) supports the argument that low inventory levels are an incentive to increase production in order to prevent running out of stock in the future. It is therefore assumed that the price based adjustment (*PA*) is supplemented by a further extension of the crop area. The inventory level is measured relative to what is currently supplied, i.e.:

$$IC(t) = \frac{IT(t)}{S(t)} \quad (10)$$

IC(t) denotes the *inventory coverage*, which indicates for how many time periods the current total inventory *IT(t)* could serve the supply *S(t)*. If *IC(t)* falls below a desired level an additional extension of the crop area takes place. Figure 5 illustrates the relationship used to model this context. The graph shows that below a desired inventory coverage level (*dIC*) an adjustment factor between 0 and *m* is computed.

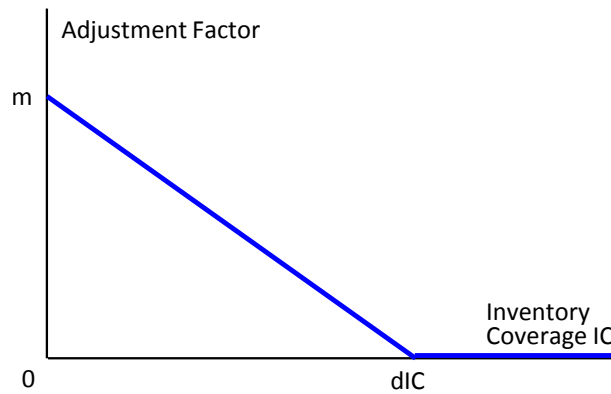


Figure 5: Inventory based adjustment of the crop area

From this we derive the inventory based adjustment of the crop area *IA(t)* as

$$IA(t) = \text{Max} \left[\left(m - \frac{m}{dIC} IC(t) \right), 0 \right] CA(t) \quad , \quad m, dIC > 0 \quad (11)$$

where the *Max[·]* operator assures that the additional acreage is always positive or zero, and the crop area can only decline due to low price expectations.

The annual production is related to the crop area via the harvest function *H(t)*:

$$H(t) = y CA(t - T_p) \quad , \quad y > 0 \quad (12)$$

The expression *CA(t - T_p)* represents the crop area lagged by *T_p* time units to account for the production period, and *y* specifies the yield per unit of crop land.

3 Model Results

The model was implemented in © Vensim and solved using a 4th order Runge-Kutta integrator. It was simulated using various parameter settings in order to explore its potential to generate different dynamic patterns. The simulation runs revealed that the model is able to generate a market equilibrium as well as persistent price fluctuations.

Three model runs are presented which highlight important factors affecting the dynamic properties of the system. The essential parameter settings of these runs are delineated in Table 1. For the ‘base run’, the model was calibrated so it terminates at a market equilibrium after an initial deviation. The remaining two simulation runs were designed to reveal factors leading to persistent price fluctuations. In the second run, the demand level given by the parameter b in equation (2), was increased by 25 %. In the last model run, the inventory based adjustment of production was introduced by setting the desired inventory coverage (i.e. parameter dIC in equation 11) at 1.5 years, with all other parameters remaining the same as in the base run.

Table 1: Parameter settings of the simulation runs

	Equation number	Base run (equilibrium)	Increased demand level	Inventory based adjustment of production
Demand level (b)	2	1000	1250	1000
Demand elasticity (c)	2	0.25	0.25	0.25
Long term expectation adjustment time (τ)	3	2	2	2
Short term expectation adjustment time (τ)	3	0.5	0.5	0.5
Desired inventory coverage (dIC)	11	-	-	1.5

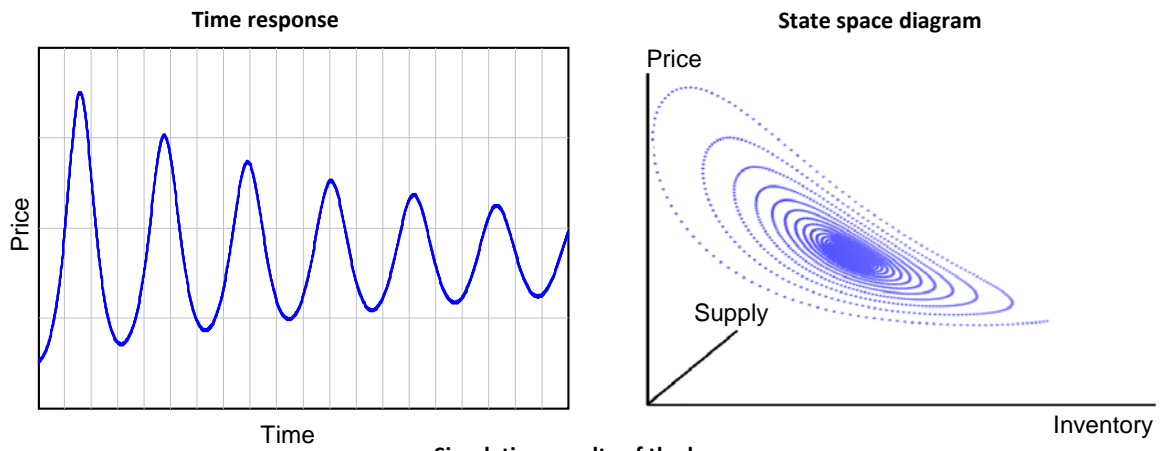
The simulation results are depicted in Figure 6. The graphs on the left hand side of the Figure represent the time response, while the graphs of the right hand side portray the primary state variables of the model, i.e. price, supply and total inventory in three-dimensional space. The price series generated by the base run of the model (part a. of Figure 6) exhibits damped cycles which ultimately converge at the equilibrium price. This can also be seen from the corresponding state space diagram. The trajectory of the three state variables spirals inwards and finally collapses at the market equilibrium, regardless of the starting point. The system is globally stable and its dynamic behaviour can be characterized as *point attractor*. This response is the same as usually generated by competitive storage models based on rational expectations. Persistent price fluctuations can only occur in reaction to repeated external shocks.

Part b. of Figure 6 depicts the simulations results if the demand level (parameter b) is augmented by 25 %. The demand increase (with all other parameters remaining the same as in the base run) dramatically changes the dynamic properties of the system. The generated price series exhibits persistent fluctuations. The corresponding state space diagram delineates a so called *limit cycle*. Regardless of the starting point, all trajectories converge on one orbit. Since the model is completely deterministic the revealed price volatility is endogenous and the cycling emerges without external shocks.

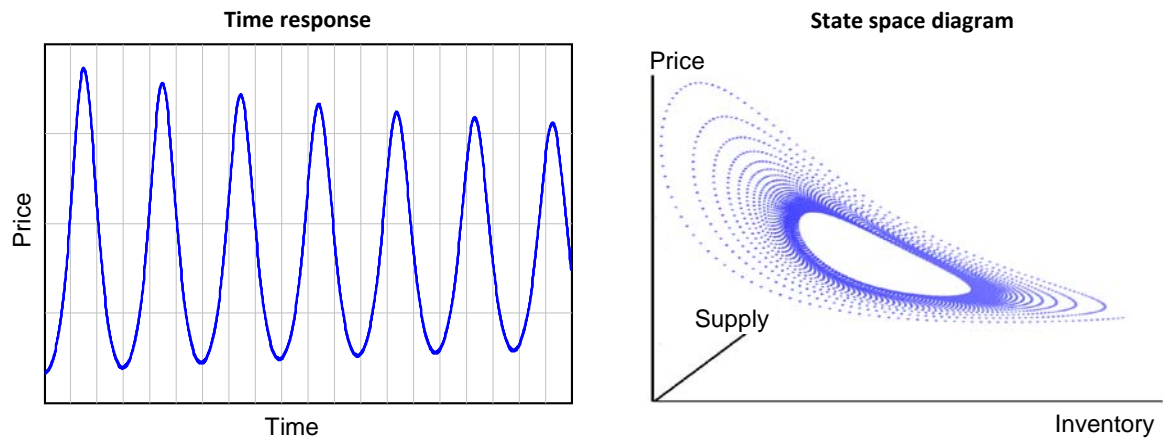
This response is primarily caused by the price dependent storage time (equation 5) and triggered by the demand increase. If the behaviour proposed in the model corresponds to that of real world actors, a sharp demand increase could cause a complete change of the dynamic properties of the system. If this happened in reality, we would possibly diagnose a significant volatility increase.

In the first two model runs, a particular reaction to low inventory levels was not yet considered. This has been changed in the third simulation experiment. With all other parameter equal to the base run, the desired inventory coverage (dIC) is now set to 1.5 years. The simulation results given in part c. of Figure 6, again indicate a change of the dynamic properties. As in the second model run, the simulated attractor is a *limit*

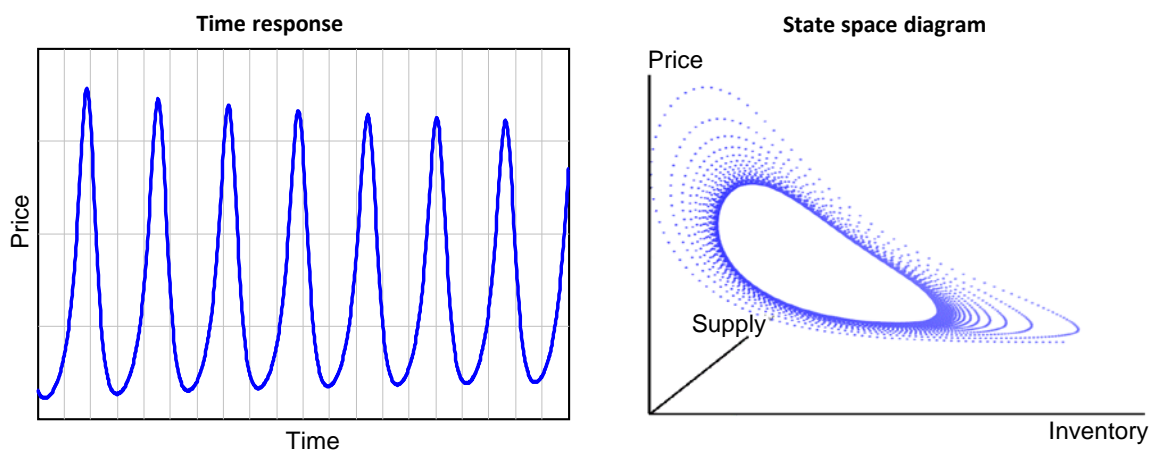
cycle. Thus, with the proposed reaction to low inventory levels the model reveals another possible cause of endogenous price fluctuations and volatility increase.



a. Simulation results of the base run



b. Simulation results at 25 % demand increase



c. Simulation results caused by inventory based adjustment of production

Figure 6: Simulation Results

4 Conclusions

A *system dynamics* modelling approach was applied to identify possible causal factors of persistent price fluctuations on agricultural commodity markets. The analysis showed that plausible behaviour of actors in the market can lead to persistent price fluctuations, even in the absence of external shocks. The dynamic properties of the system are due to inherent nonlinearities along with the built in time lags. These nonlinearities refer primarily to (1) the price adjustment process, (2) the price dependent delay governing the short term stock management and (3) the inventory based adjustment of the cultivated area. The latter induces a production increase if the current stock falls below a certain level. Only if this reaction is eliminated and the demand level is generally low (relative to the production potential), the system converges at a stable market equilibrium. Otherwise the simulation results reveal permanent endogenous fluctuations in form of a limit cycle.

If this result mirrors the conditions of real commodity markets it has important practical implications, as many policy measures aimed at price stabilisation are likely to fail under such circumstances. *System dynamics models* can help in revealing important nonlinearities and their implications. However, the model presented in this paper is still hypothetical and not yet validated. The calibration as well as the validation of system dynamics models is generally difficult, since direct econometric estimation is mostly impossible. A promising methodology is the application of *nonlinear time series analysis* along with *phase space reconstruction* to identify an attractor from empirical time series data in a first step, and subsequently develop a theory-based structural model of the industry that matches the properties of the reconstructed attractor. This approach was used by Berg and Huffaker (2015) to investigate causal factors driving the German hog-price cycle. Its main difficulty is that it requires large amounts of data, i.e. long time series which are not always available or contain severe structural breaks. Alternatively, one could try to validate behavioural hypotheses by means of *economic experiments*. Arrango and Moxnes (2012) used this methodology to study the impacts of market complexity on the behaviour of agents. Either way appears promising to advance the analysis of dynamic systems in general, and the analysis of commodity market dynamics in particular.

References

- Arango, S., Moxnes, E. (2012). Commodity cycles, a function of market complexity? Extending the cobweb experiment. *Journal of Economic Behavior & Organization* 84(1), pp 321–334.
- Berg, E., Huffaker, R. (2015). Economic Dynamics of the German Hog-Price Cycle. *Int. J. Food System Dynamics* 6(2), pp 64–80.
- Cafiero, C., Bobenrieth, E. S. A., Bobenrieth, J. R. A., Wright, B. D. (2011). The empirical relevance of the competitive storage model. *Journal of Econometrics* 162(1), pp 44–54.
- Carter, C. A., Rausser, G. C., Smith, A. (2011). Commodity booms and busts. *Annual review of resource economics* 3, pp 87–118.
- Chavas, J.-P., Holt, M. T. (1993). Market instability and nonlinear dynamics. *American Journal of Agricultural Economics* 75(1), pp 113–120.
- Day, R. (1982). Complex Behaviour in System Dynamics Models. *Dynamica* 8 (Part II Winter), pp 82–89.
- Deaton, A., Laroque, G. (1992). On the behaviour of commodity prices. *The review of economic studies* 59(1), pp 1–23.

- Deaton, A. S., Laroque, G. (1995). Estimating a nonlinear rational expectations commodity price model with unobservable state variables. *Journal of applied econometrics* 10, pp S9-S40.
- Deaton, A. S., Laroque, G. (1996). Competitive storage and commodity price dynamics. *The journal of political economy* 104(5), pp 896-923.
- Ezekiel, M. (1938). The Cobweb Theorem. *The Quarterly Journal of Economics* 52(2), pp 255–280.
- Gouel, C. (2012). Agricultural price instability: A survey of competing explanations and remedies. *Journal of economic surveys* 26(1), pp 129–156.
- Guerra, E. A., Bobenrieth H., E. S. A., Bobenrieth H., J. R. A., Cafiero, C. (2015). Empirical commodity storage model: The challenge of matching data and theory. *European Review of Agricultural Economics* 42(4), pp 607–623.
- Gustafson, R. L. (1958). Carryover levels for grains: A method for determining amounts that are optimal under specified conditions. Washington: U.S. Dept. of Agriculture.
- Irwin, S. H., Thraen, C. S. (1994). Rational expectations in agriculture? A review of the issues and the evidence. *Review of Agricultural Economics* 16, pp 133–158.
- Mackey, M. C. (1989). Commodity price fluctuations: Price dependent delays and nonlinearities as explanatory factors. *Journal of economic theory* 48, pp 497–509.
- Miranda, M. J., Glauber, J. W. (1993). Estimation of dynamic nonlinear rational expectations models of primary commodity markets with private and government stockholding. *The review of economics and statistics* 75, pp 463–470.
- Mitra, S., Boussard, J.-M. (2012). A simple model of endogenous agricultural commodity price fluctuations with storage. *Agricultural economics* 43(1), pp 1–15.
- Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica* 29(3), pp 315–335.
- Wright, B. D. (2011). The Economics of Grain Price Volatility. *Applied Economic Perspectives and Policy* 33(1), pp 32–58.
- Wright, B. D., Williams, J. (1982). The economic role of commodity storage. *Economic Journal* 92, pp 596–614.