Available online at www.centmapress.org

INTERNATIONAL JOURNAL ON FOOD SYSTEM DYNAMICS

Int. J. Food System Dynamics 13 (2), 2022, 225-246

DOI: https://dx.doi.org/10.18461/ijfsd.v13i2.B8

Optimizing Vendor-Buyer Inventory Model with Exponential Quality Degradation for Food Product Using Grey Wolf Optimizer

Dana Marsetiya Utama, Sri Kurnia Dwi Budi Maulana, Teguh Baroto, and Shanty Kusuma Dewi

University of Muhammadiyah Malang, Jl. Tlogomas No. 246, 65144 Malang, East Java, Indonesia.
dana@umm.ac.id, kurniamaulana2@gmail.com, teguh@umm.ac.id, shanty@umm.ac.id

Received August 2021, accepted February 2022, available online April 2022

ABSTRACT

Inventory is an essential factor in the supply chain. Inventory problems are increasingly complex for perishable products such as food. This study proposes a Single Vendor-Single Buyer (SVSB) model for food products by considering exponential quality degradation. The objective function of this problem is to maximize the Joint Total Profit (JTP) of the SVSB system. The frequency of ordering raw materials (m), the frequency of delivery of the finished product (n), and the time of the inventory cycle (T) were the three (3) decision variables introduced in the study. This study proposes the Grey Wolf Optimizer (GWO) algorithm as an optimization tool for SVSB problems. A case study was conducted on a food company in Indonesia. Sensitivity analysis on costs, revenue, and JTP was also presented. The results showed that raw materials' quality degradation level affected JTP. The results also suggested that the GWO algorithm performs better than the Genetic Algorithm (GA) to optimize the SVSB inventory model.

Keywords*: Optimization Inventory; Grey Wolf Optimizer; vendor-buyer; inventory model*

1 Introduction

Supply chain performance has become a determining factor for companies to challenge competition globally (Ibrahim, Putri, and Utama, 2020). Supply chain management is an integrated approach in managing information, finances, and goods from vendors to end customers (Mentzer et al., 2001; Gholami and Mirzazadeh, 2018; Utama, Santoso, Hendrawan, and Dania, 2022). One of the critical factors in the supply chain is inventory management (Goyal, Deshmukh, and Control, 1997) (Kumar and Kumar, 2017) (Gholami and Mirzazadeh, 2018) (Utama, Wardani, Halifah, and Pradikta, 2019). Inventory is an essential asset in a company that plays a vital role in the smoothness of production (Maulana et al., 2019; Utama, Widodo, Ibrahim, Hidayat, and Dewi, 2020; Widodo and Utama, 2019). Moreover, the inventory problem significantly impacts the company's total profit (Utama, Kholik, and Mulya, 2020) (Pando, San-José, and Sicilia, 2020). One of the critical problems in the supply chain system is integrating inventory decisions between vendors and buyers (Huang, 2004). Although several vendor-buyer supply chain management strategies have been proposed, inventory management policies have proven can improve supply chain and company performance (Sarmah, Acharya, and Goyal, 2006). The problem of inventory on vendor-buyer has variations such as Single Vendor-Single Buyer (SVSB) (Huang, 2004) and Single Vendor-Multi Buyers (SVMB) (Sarkar, Majumder, Sarkar, Kim, and Ullah, 2018) (Tarhini, Karam, and Jaber, 2019) (Agustiandi, Aritonang, and Rikardo, 2021). In the last decade, there has been increasing interest in the SVSB inventory problem (Giri, Dash, and Sarkar, 2020).

Goyal (1988) was the first researcher that introduces the SVSB Inventory model. Since then, many SVSB issues have been investigated. Several studies on the SVSB Inventory model have been proposed to solve Inventory problems. Hill (1999) proposed an economic production quantity model for the SVSB. Furthermore, this model was developed to determine the production and shipment policy (Hill and Omar, 2006). Yao, Evers, and Dresner (2007) constructed a vendor-managed inventory model by considering continuous replenishment and just-in-time purchasing. The SVSB model for multi-product multi-constraints was proposed by Pasandideh, Niaki, and Nia (2011) and Sadeghi, Sadeghi, and Saidi Mehrabad (2011). Their research utilized genetic algorithms for problem optimization. Zanoni and Zavanella (2007) developed a model that integrates the transport-inventory system. They proposed a heuristic procedure to solve the problem. The SVSB model considering deteriorating and defective items have also been developed by Lee and Kim (2014).

Furthermore, Liu, Li, and Yang (2019) suggested an SVSB model with deteriorating items. They offered a heuristic procedure to solve the problem. Sekar and Uthayakumar (2018) developed a model involving multiple production setups and rework. A model considering lead times and stochastic demand was researched by Mou, Cheng, and Liao (2017). In addition, Vijayashree and Uthayakumar (2017) also offered a model with ordering cost reduction dependent on lead time. The model considering learning effect, fuzzy demand, and imperfect quality was projected in Fu, Chen, and Sarker (2019). They proposed an algorithm heuristic to solve this problem. AlDurgam, Adegbola, and Glock (2017) developed a model with stochastic demand and variable production rates. A heuristic solution procedure was offered to solve this problem.

Previous research from the SVSB model stated that most of the models developed did not consider quality degradation. Only a few papers have discussed deteriorating and defective items. Some of these studies were investigated by Lee and Kim (2014), Liu et al. (2019), and Fu et al. (2019). Unfortunately, their research was not suitable for the problem of food products. Food products have unique characteristics because they are included in the perishable product (Lee, Fauza, Amer, and Prasetyo, 2014; Rau, Wu, and Wee, 2004; Muhammad Faisal Ibrahim, Mardhiyyah, Rusdiansyah, Boer, and Utama, 2020). This product undergoes rapid quality degradation (Gusti Fauza, Amer, Lee, and Prasetyo, 2016; Ouyang, Wu, and Yang, 2006). Improper inventory management affects supply chain performance (Blackburn and Scudder, 2009). The right decision in vendor-buyer inventory management can increase the company's profit (Gusti Fauza, Amer, and Lee, 2013; Fauza, Amer, Lee, and Prasetyo, 2015; Fauza, Prasetyo, and Amanto, 2018). Food products have linear and exponential quality degradation characteristics (Wang and Li, 2012; Yang and Tseng, 2015). We noted only Gusti Fauza, Prasetyo, Dania, and Amanto (2018) research discussed SVSB for food products. Unfortunately, their research assumes that the degradation of raw material quality is linear. In addition, the demand for raw materials was assumed to be the same as the demand for finished products. This assumption is not realistic in actual conditions. In fact, some food products have exponential quality degradation characteristics. Moreover, the demand for finished products is not the same as the demand for raw materials.

Based on previous research, no study SVSB inventory model considers exponential quality degradation. Therefore, this study develops an SVSB inventory model to maximize joint total profit by considering exponential quality degradation. The proposed model is developed from Gusti Fauza et al. (2018) with some development. First, we developed a model with exponential quality degradation for raw material. Second is, this research considers the conversion coefficient of the finished product to raw material. Then, a sophisticated Grey Wolf Optimizer (GWO) algorithm is offered as an optimization tool. To maximize joint total profit, adequate procedures are needed to optimize SVSB problems (Chang, Teng, and Goyal, 2010). One of them is a popular meta-heuristic procedure for the optimization of the vendor-buyer inventory model problem with a Genetic Algorithm (GA) (G Fauza et al., 2018; Gusti Fauza et al., 2018; Pasandideh et al., 2011; Sadeghi et al., 2011). Other meta-heuristic algorithms are also proposed for vendor buyer model optimization, such as Particle Swarm Optimization (PSO) (Taleizadeh, Niaki, Shafii, Meibodi, and Jabbarzadeh, 2010), and a combination of PSO and GA (Sadeghi, Mousavi, Niaki, and Sadeghi, 2013). Although several metaheuristic methods have been proposed, unfortunately, no studies discuss the computational time in solving SVSB problems. Therefore, this study also analyzes the computational time to solve the SVSB problem.

The motivations of this research are described as follows: (1) to the best of the author's knowledge, the problem of the decision to integrate the SVSB inventory model by considering exponential quality degradation for food products has never been investigated. In addition, several previous research models assumed that the total demand for raw materials at the vendor level was the same as the demand at the buyer level. This study develops an SVSB model with exponential quality degradation of raw materials. The proposed model also considers the amount of raw material demand at the vendor level, which is not the same as the demand at the buyer level. The second research motivation is that no previous research has used GWO to optimize SVSB problems. The GWO algorithm is a sophisticated algorithm inspired by Grey Wolf in finding food in nature. It proved effective compared to the GA algorithm (Mirjalili, Mirjalili, and Lewis, 2014). This algorithm has been successfully applied to scheduling problems (Utama, 2021; Jiang and Zhang, 2018), and feature selection (Al-Tashi, Kadir, Rais, Mirjalili, and Alhussian, 2019), prediction (Wei et al., 2017). It is hoped that this research contributes to a deeper understanding of the problem of the SVSB inventory model for food products. The contribution of this research is described as follows: (1) proposing an integrated SVSB inventory model by considering quality degradation exponentially for food products, and (2) proposing a sophisticated GWO procedure as an optimization tool for SVSB problems.

The composition of this paper is presented as follows; Section 2 presents the literature review. System characteristics, assumptions, notations, and mathematical models are described in Section 3. Section 4 presents the proposed GWO algorithm to optimize the SVSB inventory model. Section 5 contains data and experimental procedures. Optimization of the SVSB model with GWO, sensitivity analysis, and comparison of algorithms are presented in section 6. The final section of this paper displays the conclusion and suggestions for further research.

2 Literature Review

The literature review on the topic of SVSB is discussed in this section. Some researchers have published SVSB research. The SVSB problem was first studied by Banerjee (1986), with lot size as a decision variable. In this investigation, the vendor served as a manufacturer. Suresh K Goyal (1988), Goyal and Gupta (1989), Hill (1999), and Lu (1995) also studied the same topic. In addition, Hill and Omar (2006) established this model to decide the policy of production and shipment. The lead time was taken into account by Ben-Daya and Raouf (1994). Ouyang, Yeh, and Wu (1996) established this model, including lost sales and backorders. Yao et al. (2007) offered a model that incorporates continuous replenishment and just-in-time purchase. The SVSB model for multi-product multi-constrain instances was proposed by Sadeghi et al. (2011) and Pasandideh et al. (2011). They employed GA to optimize the problem in their research. Sekar and Uthayakumar (2018) presented a concept incorporating rework and setup. In addition, Mou et al. (2017) created a model that considered stochastic demand and lead times. A new model with stochastic demand and variable production rates was created by AlDurgam et al. (2017). Vijayashree and Uthayakumar (2017) proposed a model in which ordering costs are reduced in proportion to lead time. Zanoni and Zavanella (2007) developed a model that considers the transportation and inventory systems.

There have also been several studies on the SVSB model with imperfect quality. The SVSB model was projected by Lee and Kim (2014) by taking deteriorating and defective items into account. The SVSB model proposed by Liu et al. (2019) has a decreasing production and shipment policy item. Fu et al. (2019) also proposed a model that considers imperfect quality, fuzzy demand, and the learning effect. Quality degradation research is scarce, according to previous studies. Only Gusti Fauza et al. (2018), Liu et al. (2019), Fu et al. (2019), and Lee and Kim (2014) discussed deteriorating and defective items. There are only a few products on the market that can be described in such a way. Food products degrade linear and exponentially (Wang and Li, 2012; Yang and Tseng, 2015). Only Gusti Fauza et al. (2018) discussed the SVSB model for food products, and we are unaware of any other studies on the subject. Assumed in their study, the degradation of raw material quality is a straight line. Untimely delivery was recently proposed by Çömez-

Dolgan, Moussawi-Haidar, and Jaber (2021). Herbon (2021) provided a model that takes into account a production cycle length that is constrained.

Table 1 shows the comparison of this study with previous SVSB studies. It demonstrates that the objective function of cost minimization dominates the SVSB problem. The objective function of Maximize Profit, on the other hand, is still rarely investigated. Furthermore, this study involves a quality degradation in raw material and a shelf-life-based price function for finished products at the buyer level. In raw material, exponential degradation is considered in this model. The GWO algorithm is proposed as a sophisticated procedure to solve this problem. This research also analyzes computation time which has not been discussed by previous SVSB research.

Table 1
Comparison of studies.

Author	Quality degradation/ deteriorating	Type degra-dation	Shelf-life based price function	Com-pu-tation time	Objective function		Solution Procedure
					Minimize Total cost	Maximize Profit	
Banerjee (1986)	-	-	-	-	V	-	Exact
Suresh K Goyal (1988)	-	-	-	-	V	-	Exact
Goyal and Gupta (1989)	-	-	-	-	V	-	Exact
Ben-Daya and Raouf (1994)	-	-	-	-	V	-	Heuristic
Lu (1995)	-	-	-	-	V	-	Heuristic
Ouyang et al. (1996)	-	-	-	-	V	-	Heuristic
Hill (1999)	-	-	-	-	V	-	Heuristic
Hill and Omar (2006)	-	-	-	-	V	-	Heuristic
Yao et al. (2007)	-	-	-	-	V	-	Heuristic
Pasandideh et al. (2011)	-	-	-	-	V	-	GA
Sadeghi et al. (2011)	-	-	-	-	V	-	GA
Lee and Kim (2014)	V	-	-	-	-	V	Heuristic
Sekar and Uthayakumar (2018)	V	-	-	-	V	-	GA
Mou et al. (2017)	-	-	-	-	V	-	Heuristic
AlDurgam et al. (2017)	-	-	-	-	V	-	Heuristic
Vijayashree and Uthayakumar (2017)	-	-	-	-	V	-	Heuristic
Gusti Fauza et al. (2018)	V	Linear	V	-	-	V	GA
Liu et al. (2019)	V	-	-	-	V	-	Heuristic
Fu et al. (2019)	V	-	-	-	V	-	Heuristic
Çömez-Dolgan et al. (2021)	-	-	-	-	V	-	Heuristic
This research	V	Exponential	V	V	-	V	GWO

3 System characteristics

This section describes the characteristics of the SVSB system. The SVSB system is depicted in Figure 1. The vendor (manufacturer) orders raw material from the supplier for m times to fulfill the demand (D) of the buyer during the inventory cycle T. The raw material needed to meet the demand is λD. Raw materials at the vendor (manufacturer) level experience quality degradation during storage. The vendor processes the raw material to become a finished product with a production level of P. The finished product is sent n times to fulfill the demand (D) of the buyer. Raw materials in this system are categorized as perishable products with an exponential level of quality degradation. The finished product is a food product with an expiration date in this study. The finished product is based on three price categories: maximum, medium, and minimum. So that as the expiration date approaches, consumer interest in buying from buyers decreases. However, this can be overcome by reducing product prices (Gusti Fauza et al., 2016).

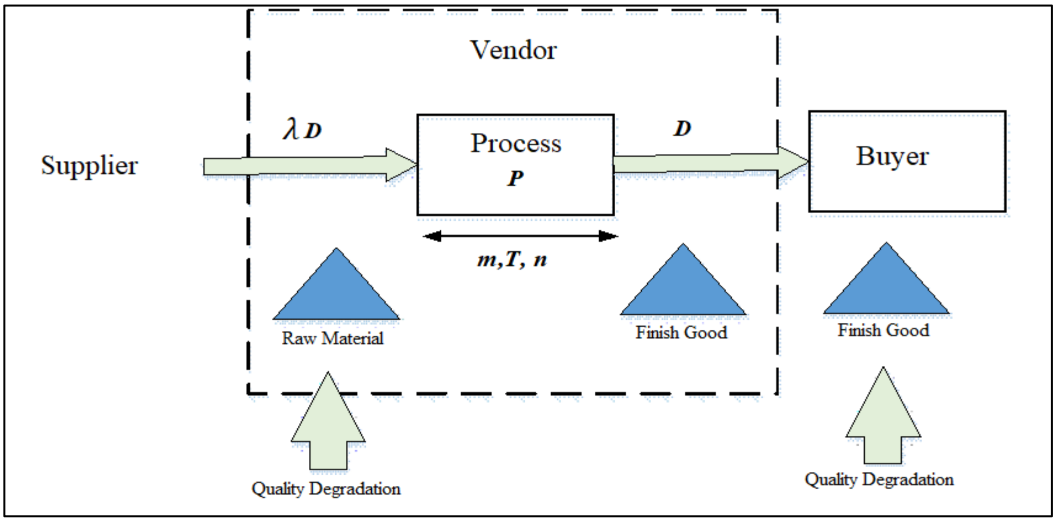

Figure 1. Single Vendor-Single Buyer System

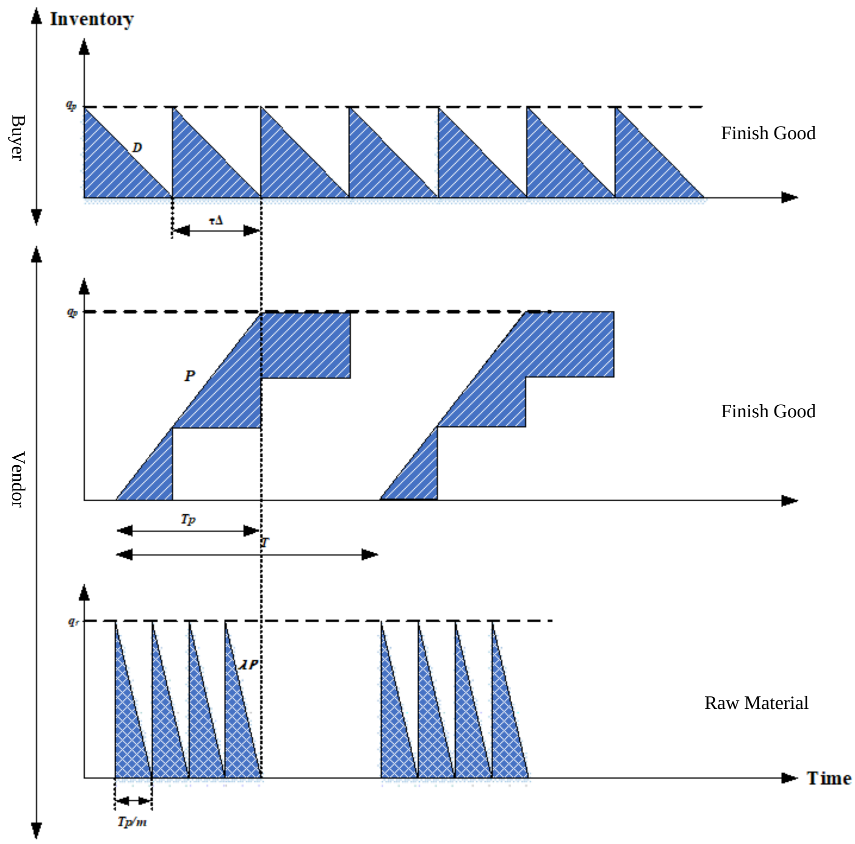

Figure 2. Inventory profile on a Single Vendor-Single Buyer system

The inventory profile in the SVSB buyer's system is described in Figure 2. Raw materials and finish goods are controlled by the inventory level for the SVSB system. There is a demand for finished products (D) at the buyer level to be fulfilled within the horizon. The finished product is shipped n times as many batches from a vendor to the buyer with a q_p/n size. Thus, the vendor delivery cycle of the finished product is q_p/D or T/n, which is then denoted as $\tau\Delta$. q_p is the quantity of the finished product to fulfill demand during the production cycle T or $q_p = DT$, then T_p is equal to DT/P. To fulfill the demand for D buyer products, the vendor performs production at a production rate of P. The raw material needed to meet the production rate is λP during the production time of T_p. The conversion value of raw materials to finished products is λ. Furthermore, vendors (manufacturers) procure raw materials from suppliers with lot size q_r where the procurement cycle is T_p/mTp. Assumptions, notations, and mathematical models in appendix A.

4 Proposed GWO Algorithm

This section describes a proposed procedure for solving the SVSB inventory model problem. The algorithm proposed to solve this problem is the Grey Wolf Optimizer (GWO) Algorithm. The GWO algorithm is a new advanced algorithm inspired by the hunting behavior of the grey wolf. This algorithm was proposed by Mirjalili et al (Mirjalili et al., 2014). The grey wolf has a unique behavior when hunting its prey. In the hunting group, the grey wolf divides itself into four types of roles, namely alpha (α), beta (β), delta (δ), and omega (ω). The α wolf is the lead wolf, where α is a wolf who can be the decision-maker in the group. The beta β wolf is a wolf advising the α wolf in terms of decision making. The delta (δ) wolves are subordinate wolves of the α and β with the role of helping to hunt, guarding the boundaries, and protect the groups. The ω wolf group is the wolf that causes mistakes.

The grey wolves have 3 phases in hunting prey: tracking, siege, and attacking stages. From this behavior, tracking and siege phases are formulated by the following Equations (1), (2), (3), and (4):

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_p(it) - \vec{X}(it)\right| \tag{1}$$

$$\vec{X}(it+1) = \vec{X}_p(it) - \vec{A} \cdot \vec{D} \tag{2}$$

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \tag{3}$$

$$\vec{C} = 2 \cdot \vec{r}_2 \tag{4}$$

Where looping (iteration) in GWO denotes by it. $\vec{X}_p$ is the vector of the prey position, and $\vec{X}$ represents the position vector of the grey wolf. $\vec{D}$, $\vec{A}$, and $\vec{C}$ represent the vector coefficients. The behavior of capturing prey is indicated by the value of a decreasing linearly from 2 to 0 during iteration. The $r1$ dan $r2$ are random vectors with values from 0 to 1.

The hunt is led by α, while β and δ occasionally hunt. Therefore, α is the first best candidate solution, β second, and δ third. In finding the optimal position, hunting is represented in Equations (5), (6), and (7).

$$\vec{D}_\alpha = \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}\right|, \vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}\right|, \ \vec{D}_\delta = \left|\vec{C}_3 \vec{X}_\delta - \vec{X}\right| \tag{5}$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot (\vec{D}_\alpha), \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta), \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta) \tag{6}$$

$$\vec{X}(it+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{7}$$

The GWO algorithm proposed by Mirjalili et al. (2014) is an algorithm to solve continuous problems. As described in section 2, the SVSB inventory model problem was categorized as a mixed problem. For this reason, it was more appropriate if three decision variables were used in this problem. Two decision variables: frequency of raw material ordering (m) and delivery of finished products to buyers (n), were the integers. Furthermore, the time decision variable during the Inventory cycle (T) was continuous. Therefore, three (3) dimensions (decision variables) were utilized in GWO to optimize the SVSB inventory model. For the conversion from a continuous number to an integer, this study proposed rounding for the variables m and n. An illustration of the conversion of decision variables is presented in Figure 3. If the decimal values of m and n were above 0.5, then the m and n were rounded up.

Conversely, if the decimal values for m and n were below 0.5, the values were rounded down. To optimize the SVSB inventory model, the data inputted included production data, demand data, cost data, and quality data. The required initialization of GWO data included the number of wolves or search agents. The dimensions (decision variables) used were the three (m, n, T), the maximum iteration, and the upper and lower limits for the decision variables. The fitness value at GWO was based on the JTP value, which was calculated based on Equation (8). GWO's pseudo code for the SVSB inventory model is shown in Algorithm 1.

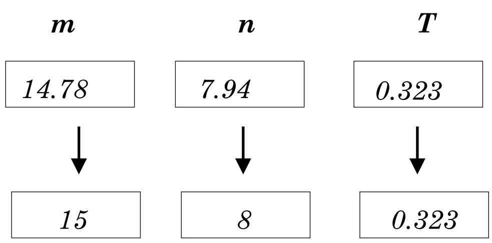

Figure 3. Illustration of Decision Variable Conversion.

Table 2

Algorithm 1. GWO Pseudo-code for SVSB Inventory Model.

Input IPP data: demand, production rate, costs, quality
Initialize the GWO data: Population, Dimension, max iteration, upper bound, and lower bound
Initialize a, A, and C to get the position (m, n, T)
Calculate the fitness of each search agent (JTP)
$X\alpha$ = the best search agent
$X\beta$ = the second-best search agent
$X\delta$ = the third best search agent
***While** (t < Max number of iterations)*
    ***for** each search agent*
        Update the position of the current search agent (update m, n, T)
    end for
    Update a, A, and C
    Calculate the fitness of all search agents
    Update $X\alpha$, $X\beta$, and $X\delta$
    t = t + 1
end while
return $X\alpha$

5 Data and Experimental Procedure

5.1 Data

The research data were based on case study data of food companies in Indonesia. The research data are presented in Table 3.

Table 3

Research Data

Parameters	Value	
P	1,418	kg/month
D	1,298	kg/bulan
λ	7.2	
Closs	7,200	IDR/liter
Csale	85,000	IDR/kg
Cr	7,200	IDR/liter
Cp	550	IDR/kg
Ar	50,000	IDR/delivery
Ap	50,000	IDR/Delivery
Sp	60,000	IDR/Setup
Hr	520	IDR/(liter/month)
Hp	528	IDR/(kg/month)
pmax	105,000	IDR/kg
pmin	0	IDR/kg
k	0.5	-
τ_{sl}	8	month
τ_{Start}	6	month

5.2 Experimental Procedure

In this section, this study describes the experimental procedure in optimizing SVSB inventory. The GWO parameters used in optimizing the SVSB inventory model are presented in Table 3. The optimization results using the GWO algorithm were used to test the sensitivity analysis of the SVSB model. The variables used in the sensitivity analysis were the frequency of raw material orders (m), the frequency of delivery of the finished product (n), the inventory cycle time (T), the rate of quality degradation of raw materials (k), and the conversion coefficient of the finished product to raw materials (λ). Analysis of the effects of m and n on costs and the revenue and JTP was carried out 21 times. The value of the decision variables m and n was changed from 1 to 100. The effect of T on costs, revenue, and JTP was tested 20 times. The decision variable T was changed from a value of 0.05 to 1. The effect of the rate of quality degradation of raw materials *(k)* on cost and profit was shifted by nine experiments. The value of k was changed from a value of 0.1 to 0.9. The effect of λ on costs, revenue, and JTP was carried out 14 times. All sensitivity analysis trials were recorded as JTP, TC_{rm}, TC_{pm}, TC_{pr} and JTR.

To test the performance of the GWO algorithm, this study also compared it with GA. Comparisons were made with ten different experimental data demands with a range of 1000 – 1350. Each experiment was recorded *JTP* and Computation Time. Furthermore, to evaluate the algorithm's performance, we used the percentage of the algorithm gap solution on the optimal solution (Equation 8) (D. M. Utama, Widodo, Ibrahim, and Dewi, 2020) and the Solution Ratio (SR) (Equation 9) (Dewi and Utama, 2021). Finally, the JTP and Computation Time results from GWO and GA were statistically tested using the Wilcoxon Test, which was run using SPSS 21.

$$Gap_{Solution} = \frac{|Solution_{Algorithm} - Solution_{Optimal}|}{Solution_{Optimal}} \times 100\% \tag{8}$$

$$SR = \frac{Solution_{algorithm}}{Solution_{Optimal}} \times 100\% \tag{9}$$

The GA algorithm parameters used as benchmarks are presented in Table 4. The GA parameters used were based on research conducted by Fauza et al. (G. Fauza, Y. Amer, S. H. Lee, and H. Prasetyo, 2015). The GWO algorithm was compared with the GA algorithm to determine the JTP on the SVSB inventory model problem. In addition, to determine the performance of the GWO and GA algorithms, the total profit generated in each iteration was recorded. The algorithm performance was then compared. The comparison was done using convergence analysis on both algorithms. The convergence curve was formed by moving the solution value from iteration to iteration towards a concentrated point (van den Bergh and Engelbrecht, 2006). This experiment was run using the Matlab 2018a software on Windows 10 Intel i7 RAM 8 GB.

Table 4
GA benchmark algorithm parameters

Parameters	**Value**
Population numbers	100
Iteration maximum	100
Dimension	3
Upper Bound	[100 100 1]
Lower Bound	[1 1 0]
Crossover Rate	0.7
Mutation	0.2

6 Results and Discussion

6.1 SVSB Optimization using GWO

The optimization results of the SVSB and GWO problems are presented in Table 5. The results indicate that the optimal JTP value was 66,029,518 IDR. The optimal decision variable for the frequency of raw material delivery (m) was 12 times, the frequency of delivery of the finished product (n) was four times, and the production cycle time (T) was 0.76 months.

Table 5

Optimization Results with GWO

GWO Algorithm	Decision Variable		
	m	*n*	*T*
	12	4	0,76
	Vendor	Buyer	*Total*
Total Cost (IDR)	69,932,216	110,658,266	180,590,482
Revenue (IDR)	110,330,000	136,290,000	246,620,000
Total Profit (IDR)	40,397,784	25,631,734	**66,029,518**

6.2 Sensitivity Analysis

Based on the optimization results with GWO, the optimal raw material delivery frequency (m) was 12 times. With other decision variables that have not been changed, the results of 21 trials of changes in m on cost and profit are presented in Appendix Table B-1. The analysis results showed that m does not affect $TCpm$, $TCpr$, and JTR. However, *m* affects *TCrm* and *JTP*. When the *m* value was increased and decreased from the optimal value (12), the resulting *TCrm* became greater. Thus, the *JTP* system of *SVSB* was getting smaller.

The experiment results on the effect of *n* on cost and profit are presented in Appendix Table B-2. The analysis results projected that *n* does not affect *TCrm* and *JTR*. However, *n* affects $TCpm$, $TCpr,$ and JTP. When the value of *n* was increased and decreased from the optimal value (4), the issued *TCpr* was greater, and the resulting *TCpm* was smaller. Furthermore, the value of *n* was increased and derived from the optimal value resulting in a smaller *JTP* for the SVSB system.

The results of the experiment on the effect of *T* on cost and profit are presented in Appendix Table B-3. The results of the analysis suggested that *T* does not affect *JTR*. However, *T* affects$TCpm$, $TCpr$, $TCrm,$ and JTP. When the *T* value was increased and decreased from the optimal value (0.76), the resulting Cpm, $TCpr$, and $TCrm$ were greater. Consequently, the SVSB system's *JTP* value was getting smaller.

The experimental results of *k* on cost and profit are presented in Appendix Table B-4, and the experimental results of the effect of λ on cost and profit are presented in Appendix Table B-5. The experimental results indicated that *k* and λ do not affect *TCpm, TCpr,* and *JTR*. However, *k* and λ affect *TCrm* and *JTP.* When the values of *k* and λ were increased, the issued *TCrm* was greater. Consequently, the SVSB system's JTP value was getting smaller.

6.3 Computational complexity

The computational complexity of any metaheuristic algorithm should be minimized so that real-world optimization problems can be solved more quickly. Consequently, it is critical to examine the computational complexity of any search algorithm to determine its effectiveness in solving optimization problems.

It is possible to calculate the worst-case computational time for GWO's stepwise computational complexity: First, algorithms have the complexity of O(3.N), where 3 describe decision variables in the SVSB problem. N is the number of grey wolves in the population in initialization. Algorithm leaders can be chosen in O(3.N) computational effort by performing the linear search to find the leaders.

After the while loop, algorithms begin their main computations. It takes O(3.N) time to update each wolf's position in the original GWO, so the computational complexity is O(3.N). The computational complexity increases by the maximum number of iterations because updating the wolf continues until the maximum number of iterations is not reached. Alpha wolf can be selected in O(3.N) computational effort in the final step of the algorithm. GWO's total computation complexity is O(3.N+3.TN) after adding up all of the previously mentioned complexities.

6.4 Algorithm Comparison

The SVSB inventory model optimization results with the GA algorithm are shown in Table 6. The optimization results portrayed that the JTP for the SVSB system was 66,021,603. IDR with the frequency of raw material delivery (m) as much as 14 times, frequency of delivery of finished products (n) as much as five times, and production cycle time (T) for 0.84 months. Based on the comparisons in Table 4 and Table 10, total costs at vendors at GWO were slightly larger by 0.027% than GA. In comparison, costs at GWO buyers were slightly smaller by 0.024% from GA. The JTP generated by the GWO algorithm was more optimal by 0.012% than GA.

Table 6
Optimization Results with GA

GA	Decision Variable		
	m	*n*	*T*
	14	5	0,84
	Vendor	Buyer	*Total*
Total Cost (IDR)	69,913,210	110,685,187	180,598,397
Revenue (IDR)	110,330,000	136,290,000	246,620,000
Total Profit (IDR)	40,416,790	25,604,813	**66,021,603**

In addition to the *JTP* comparison, this study compared the GWO and GA algorithms based on the *JTP* for each iteration. The convergence curve of the GWO and GA algorithms can be observed in Figure 4. The experimental results pinpointed that the GWO algorithm reached the convergent point in the 10th iteration. In contrast, GA reached the convergent point in the 40th iteration. Under van den Bergh and Engelbrecht (2006), the criterion for the optimal convergence rate is the velocity in reaching the convergent point. Therefore it is concluded that the performance of the GWO algorithm is better than GA.

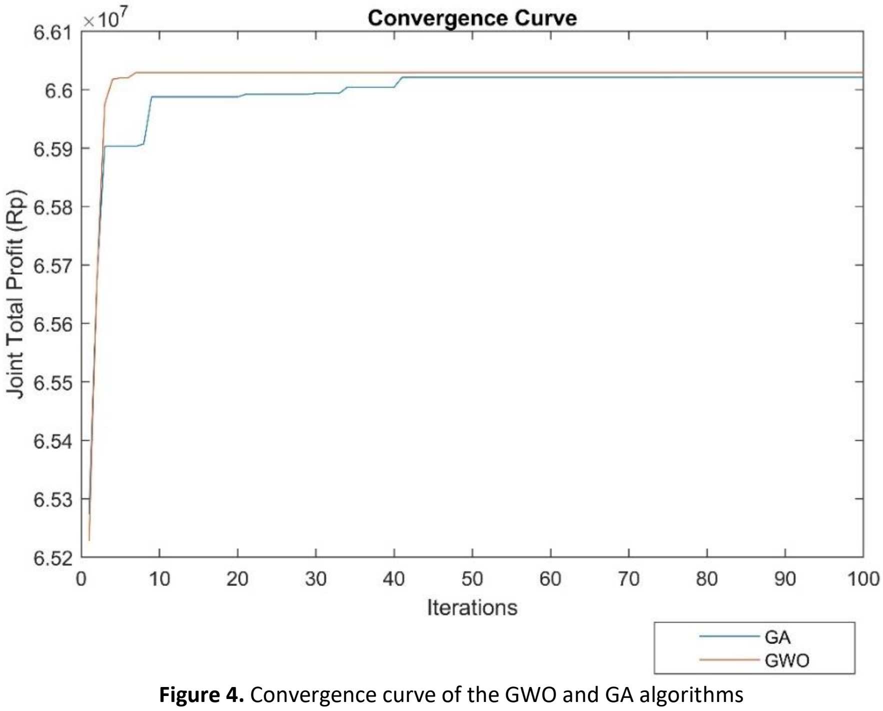

Figure 4. Convergence curve of the GWO and GA algorithms

The results of 10 trials between the GWO and GA algorithms are presented in Table 7. A percentage average gap solution of 0 percent indicates no error in the gap solution. To tell if the solution is good, the solution gap must be greater than 0. Based on Table 7, average gaps in optimal solutions for GWO and GA are 0% and 0.01%. The average gap solution shows that the GWO algorithm is more competitive than GAalgorithms.

In Table 7, SR also showed that the proposed algorithm is superior to other algorithms regarding the average percentage of solution ratio (SR). GWO and GA produce SR values of 100% and 99.97%. It shows that GWO was successful in resolving the SVSB issue.

Table 7.
Results of 10 trials between GWO and GA Algorithms

Demand (D)	JTP (IDR)		$Gap_{Solution}$ (%)		SR (%)		Computation Time (second)	
	GWO	GA	GWO	GA	GWO	GA	GWO	GA
1350	**66,076,515**	66,060,134	0	0.02	100	99.98	18.1569	90.5471
1325	**66,076,515**	**66,076,515**	0	0.00	100	100	16.5649	85.9842
1300	**66,076,515**	66,055,145	0	0.03	100	99.97	15.8937	87.3147
1298	**66,029,518**	66,021,603	0	0.01	100	99.99	19.2145	89.4712
1250	**66,057,882**	**66,057,882**	0	0.00	100	100	19.5628	82.5984
1200	**66,076,053**	66,068,162	0	0.01	100	99.99	16.7661	86.4571
1150	**66,076,053**	66,072,725	0	0.01	100	99.99	18.9469	89.5478
1100	**66,076,053**	66,074,210	0	0.01	100	99.99	20.5606	90.8475
1050	**66,076,053**	66,072,725	0	0.01	100	99.99	18.3714	87.6871
1000	**66,075,345**	66,072,725	0	0.01	100	99.99	20.8346	91.8726
	Average		0	0.01	100	99.97	18.4872	88.2328

The results of the Wilcoxon statistical test on SR are presented in Table 8. These results indicate that the proposed GWO algorithm produces an optimal solution from the Wilcoxon statistical test compared to the GA algorithm. It is evident from the value of Asymp. Sig (0.008), which is smaller than 0.05. In addition, the Wilcoxon test results for computing time are shown in Table 9. These results indicate that the proposed algorithm produces a faster computational time in solving SVSB. It is evident from the value of Asymp. Sig (0.005), which is smaller than 0.05.

Table 8.
Wilcoxon Test Results on SR

Algorithm	Mean	Std. Deviation	Minimum	Maximum	Z	Asymp. Sig.
GWO	100.0000	0.00000	100.00	100.00	-2.636	0.008
GA	99.9890	.00876	99.97	100.00		

Table 9.
Wilcoxon test results computing time

Algorithm	Mean	Std. Deviation	Minimum	Maximum	Z	Asymp. Sig.
GWO	18.4872	1.67374	15.89	20.83	-2.803	0.005
GA	88.2328	2.78488	82.60	91.87		

Conclusion

This study was objected to develop an SVSB inventory model by considering exponential quality degradation with the GWO algorithm as an optimization tool. This research has successfully developed the SVSB inventory model and GWO for optimization tools. The results showed that the rate of quality degradation of raw material *k* affected *TCrm* and *JTP*. Overall, this study concludes that the GWO algorithm has a superior performance for SVSB optimization compared to GA. Some of the limitations of this study include that GWO suffers from issues like poor local search performance and a slow convergence rate. Moreover, several assume the number of demands is fixed and constant. In future studies, improved GWO needs to be proposed to improve local search performance and convergence rate. Furthermore, uncertain demand considerations must be investigated in the next SVSB model.

Funding Statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data Availability

All data generated or analyzed during this study are included in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgment

The researcher would like to thank the University of Muhammadiyah Malang for supporting the research. In addition, the researcher would also like to extend their gratitude to the Department of Industrial Engineering Optimization Laboratory for the help of facilities.

References

Agustiandi, A., Aritonang, Y.M.K., and Rikardo, C. (2021). Integrated Inventory Model for Single Vendor Multi-Buyer with a Single Item by Considering Warehouse and Capital Constraint. *Jurnal Teknik Industri,* **22**(1): 71-84.

Al-Tashi, Q., Kadir, S.J.A., Rais, H.M., Mirjalili, S., and Alhussian, H. (2019). Binary optimization using hybrid grey wolf optimization for feature selection. *Ieee Access,* **7**: 39496-39508.

AlDurgam, M., Adegbola, K., and Glock, C.H. (2017). A single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate. *International Journal of Production Economics,* **191**: 335-350.

Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. *Decision Sciences,* **17**(3): 292-311.

Ben-Daya, M.A., Raouf, A. (1994). Inventory models involving lead time as a decision variable. *Journal of the Operational Research Society,* **45**(5): 579-582.

Blackburn, J., Scudder, G. (2009). Supply chain strategies for perishable products: the case of fresh produce. *Production and Operations Management,* **18**(2): 129-137.

Chang, C.-T., Teng, J.-T., and Goyal, S.K. (2010). Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. *International Journal of Production Economics,* **123**(1): 62-68. doi: https://dx.doi.org/10.1016/j.ijpe.2009.06.042.

Çömez-Dolgan, N., Moussawi-Haidar, L., and Jaber, M.Y. (2021). A buyer-vendor system with untimely delivery costs: Traditional coordination vs. VMI with consignment stock. *Computers and Industrial Engineering,* **154**: 107009. doi: https://doi.org/10.1016/j.cie.2020.107009.

Dewi, S.K., Utama, D.M. (2021). A New Hybrid Whale Optimization Algorithm for Green Vehicle Routing Problem. *Systems Science and Control Engineering,* **9**(1): 61-72. doi: 10.1080/21642583.2020.1863276.

Fauza, G., Amer, Y., and Lee, S.-H. (2013). *Model of an integrated procurement-production system for food products incorporating quality loss during storage time.* IACSIT-Internal Association of Computer Science.

Fauza, G., Amer, Y., Lee, S.-H., and Prasetyo, H. (2015). *An inventory model of production-inventory policy for food products considering quality loss in raw materials.* Paper presented at the 2015 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM).

Fauza, G., Amer, Y., Lee, S.-H., and Prasetyo, H. (2016). An integrated single-vendor multi-buyer production-inventory policy for food products incorporating quality degradation. *International Journal of Production Economics,* **182**: 409-417. doi: https://doi.org/10.1016/j.ijpe.2016.09.009.

Fauza, G., Amer, Y., Lee, S.H., and Prasetyo, H. (2015, 6-9 Dec. 2015). *An inventory model of production-inventory policy for food products considering quality loss in raw materials.* Paper presented at the 2015 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM).

Fauza, G., Prasetyo, H., and Amanto, B. (2018). *An integrated production-inventory model for food products adopting a general raw material procurement policy.* Paper presented at the IOP Conference Series: Materials Science and Engineering.

Fauza, G., Prasetyo, H., Dania, N.K., and Amanto, B.S. (2018). Development of food inventory model in an integrated vendor-buyer system: Case study in food industry. *AIP Conference Proceedings,* **1977**(1): 020053. doi: 10.1063/1.5042909.

Fu, K., Chen, Z., and Sarker, B.R. (2019). An optimal decision policy for a single-vendor single-buyer production-inventory system with leaning effect, fuzzy demand and imperfect quality. *Journal of Information and Optimization Sciences,* **40**(3): 633-658.

Gholami, A., Mirzazadeh, A. (2018). An inventory model with controllable lead time and ordering cost, log-normal-distributed demand, and gamma-distributed available capacity. *Cogent Business and Management,* **5**(1): 1469182. doi: 10.1080/23311975.2018.1469182.

Giri, B., Dash, A., and Sarkar, A. (2020). A single-vendor single-buyer supply chain model with price and green sensitive demand under batch shipment policy and planned backorder. *International Journal of Procurement Management,* **13**(3): 299-321.

Goyal, S., Deshmukh, S.G. (1997). Integrated procurement-production system in a just-in-time environment-modelling and analysis, **8**(1): 31-36.

Goyal, S.K. (1988). “A Joint Economic-Lot-Size Model for Purchaser and Vendor”: A Comment. *Decision Sciences,* **19**(1): 236-241. doi: https://doi.org/10.1111/j.1540-5915.1988.tb00264.x.

Goyal, S.K. (1988). “A joint economic-lot-size model for purchaser and vendor”: A comment. *Decision Sciences,* **19**(1): 236-241.

Goyal, S.K., Gupta, Y.P. (1989). Integrated inventory models: the buyer-vendor coordination. *European journal of operational research,* **41**(3): 261-269.

Herbon, A. (2021). An integrated manufacturer-buyer chain with bounded production cycle length. *Operations Research Perspectives,* **8**: 100181. doi: https://doi.org/10.1016/j.orp.2021.100181.

Hill, R.M. (1999). The optimal production and shipment policy for the single-vendor singlebuyer integrated production-inventory problem. *International Journal of Production Research,* **37**(11): 2463-2475.

Hill,R. M., Omar, M. (2006). Another look at the single-vendor single-buyer integrated production-inventory problem. *International Journal of Production Research,* **44**(4): 791-800. doi: 10.1080/00207540500334285.

Huang, C.-K. (2004). An optimal policy for a single-vendor single-buyer integrated production–inventory problem with process unreliability consideration. *International Journal of Production Economics,* **91**(1): 91-98.

Ibrahim, M.F., Mardhiyyah, Y.S., Rusdiansyah, A., Boer, M.K., and Utama, D.M. (2020). A Three-Phased Perishable Inventory Simulation Model with Quality Decrease Consideration. *Jurnal Ilmiah Teknik Industri,* ***19***(2); 198-211.

Ibrahim, M.F., Putri, M.M., and Utama, D.M. (2020). A literature review on reducing carbon emission from supply chain system: drivers, barriers, performance indicators, and practices. *IOP Conference Series: Materials Science and Engineering,* **722**: 012034. doi: 10.1088/1757-899x/722/1/012034.

Jiang, T., Zhang, C. (2018). Application of grey wolf optimization for solving combinatorial problems: job shop and flexible job shop scheduling cases. *Ieee Access,* **6**: 26231-26240.

Kumar, N., Kumar, S. (2017). An inventory model for deteriorating items with partial backlogging using linear demand in fuzzy environment. *Cogent Business and Management,* **4**(1): 1307687. doi: 10.1080/23311975.2017.1307687.

Lee, S.-H., Fauza, G., Amer, Y., and Prasetyo, H. (2014). A Vendor-Buyer Inventory Model for Food Products Based On Shelf-Life Pricing. *Operations and Supply Chain Management,* **8**. doi: https://dx.doi.org/10.31387/oscm0200146.

Lee, S., Kim, D. (2014). An optimal policy for a single-vendor single-buyer integrated production–distribution model with both deteriorating and defective items. *International Journal of Production Economics,* **147**: 161-170.

Liu, Y., Li, Q., and Yang, Z. (2019). A new production and shipment policy for a coordinated single-vendor single-buyer system with deteriorating items. *Computers and Industrial Engineering,* **128**: 492-501.

Lu, L. (1995). A one-vendor multi-buyer integrated inventory model. *European journal of operational research,* **81**(2): 312-323.

Maulana, S.K.D.B., Utama, D.M., Asrofi, M.S., Ningrum, I.S., Alba, N., Ahfa, H.A., and Zein, T.A. (2019). The Capacitated Sustainable EOQ Models: Models Considering Tax Emissions. *Jurnal Teknik Industri,* **21**(1): 12-21.

Mentzer, J.T., DeWitt, W., Keebler, J.S., Min, S., Nix, N.W., Smith, C.D., and Zacharia, Z.G. (2001). Defining supply chain management. *Journal of Business logistics,* **22**(2): 1-25. doi: https://dx.doi.org/10.1002/j.2158-1592.2001.tb00001.x.

Mirjalili, S., Mirjalili, S.M., and Lewis, A. (2014). Grey wolf optimizer. *Advances in engineering software,* **69**: 46-61.

Mou, Q., Cheng, Y., and Liao, H. (2017). A note on "lead time reduction strategies in a single-vendor-single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand". *International Journal of Production Economics,* **193**: 827-831.

Ouyang, L.-Y., Wu, K.-S., and Yang, C.-T. (2006). A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. *Computers and Industrial Engineering,* **51**(4): 637-651. doi: https://dx.doi.org/10.1016/j.cie.2006.07.012.

Ouyang, L.-Y., Yeh, N.-C., and Wu, K.-S. (1996). Mixture inventory model with backorders and lost sales for variable lead time. *Journal of the Operational Research Society,* **47**(6): 829-832.

Pando, V., San-José, L.A., and Sicilia, J. (2020). A new approach to maximize the profit/cost ratio in a stock-dependent demand inventory model. *Computers and Operations Research,* **120**: 104940. doi: https://doi.org/10.1016/j.cor.2020.104940.

Pasandideh, S.H.R., Niaki, S.T.A., and Nia, A.R. (2011). A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model. *Expert Systems with Applications,* **38**(3): 2708-2716.

Rau, H., Wu, M.Y., and Wee, H.M. (2004). Deteriorating item inventory model with shortage due to supplier in an integrated supply chain. *International Journal of Systems Science,* **35**(5): 293-303. doi: https://dx.doi.org/10.1080/00207720410001714833.

Rong, A., Akkerman, R., and Grunow, M. (2011). An optimization approach for managing fresh food quality throughout the supply chain. *International Journal of Production Economics,* **131**(1): 421-429.

Sadeghi, J., Mousavi, S.M., Niaki, S.T.A., and Sadeghi, S. (2013). Optimizing a multi-vendor multi-retailer vendor managed inventory problem: Two tuned meta-heuristic algorithms. *Knowledge-Based Systems,* **50**: 159-170. doi: https://doi.org/10.1016/j.knosys.2013.06.006.

Sadeghi, J., Sadeghi, A., and Saidi Mehrabad, M. (2011). A parameter-tuned genetic algorithm for vendor managed inventory model for a case single-vendor single-retailer with multi-product and multi-constraint. *Journal of Optimization in Industrial Engineering,* **9**: 57-67.

Sarkar, B., Majumder, A., Sarkar, M., Kim, N., and Ullah, M. (2018). Effects of variable production rate on quality of products in a single-vendor multi-buyer supply chain management. *The International Journal of Advanced Manufacturing Technology,* **99**(1): 567-581.

Sarmah, S.P., Acharya, D., and Goyal, S.K. (2006). Buyer vendor coordination models in supply chain management. *European journal of operational research,* **175**(1): 1-15.

Sekar, T., Uthayakumar, R. (2018). A production inventory model for single vendor single buyer integrated demand with multiple production setups and rework. *Uncertain Supply Chain Management,* **6**(1): 75-90.

Taleizadeh, A.A., Niaki, S.T.A., Shafii, N., Meibodi, R.G., and Jabbarzadeh, A. (2010). A particle swarm optimization approach for constraint joint single buyer-single vendor inventory problem with changeable lead time and (r, Q) policy in supply chain. *The International Journal of Advanced Manufacturing Technology,* **51**(9): 1209-1223.

Tarhini, H., Karam, M., and Jaber, M.Y. (2019). An integrated single-vendor multi-buyer production inventory model with transshipments between buyers. *International Journal of Production Economics*, **225**(4-5): 107568. doi: https://dx.doi.org/10.1016/j.ijpe.2019.107568

Utama, D. (2021). *Hybrid Grey Wolf Algorithm for Energy-Efficient Scheduling with Sequence-Dependent Setup Times: A Case Study.* Paper presented at the IOP Conference Series: Materials Science and Engineering.

Utama, D.M., Kholik, H.M., and Mulya, A.F. (2020). Integrated Procurement-Production Inventory Model with Two-Stage Production. *Jurnal Teknik Industri,* **21**(2): 185-199.

Utama, D.M., Santoso, I., Hendrawan, Y., and Dania, W.A.P. (2022). Integrated procurement-production inventory model in supply chain: A systematic review. *Operations Research Perspectives,* **9**: 100221. doi: https://doi.org/10.1016/j.orp.2022.100221.

Utama, D.M., Wardani, D.P., Halifah, S.T., and Pradikta, D.C. (2019). Model Economic Production Quantity dengan Rework Process dan Batasan Gudang. *Jurnal Sistem dan Manajemen Industri,* **3**(1): 43-49.

Utama, D.M., Widodo, D.S., Ibrahim, M.F., and Dewi, S.K. (2020). A New Hybrid Butterfly Optimization Algorithm for Green Vehicle Routing Problem. *Journal of Advanced Transportation,* **2020**: 8834502. doi: 10.1155/2020/8834502.

Utama, D.M., Widodo, D.S., Ibrahim, M.F., Hidayat, K., and Dewi, S.K. (2020). *The Sustainable Economic Order Quantity Model: A Model Consider Transportation, Warehouse, Emission Carbon Costs, and Capacity Limits.* Paper presented at the Journal of Physics: Conference Series.

van den Bergh, F., Engelbrecht, A.P. (2006). A study of particle swarm optimization particle trajectories. *Information Sciences,* **176**(8): 937-971. doi: https://dx.doi.org/10.1016/j.ins.2005.02.003.

Vijayashree, M., Uthayakumar, R. (2017). A single-vendor and a single-buyer integrated inventory model with ordering cost reduction dependent on lead time. *Journal of Industrial Engineering International,* **13**(3): 393-416.

Wang, X., Li, D. (2012). A dynamic product quality evaluation based pricing model for perishable food supply chains. *Omega,* **40**(6): 906-917.

Wei, Y., Ni, N., Liu, D., Chen, H., Wang, M., Li, Q., Cui, X., and Ye, H. (2017). An improved grey wolf optimization strategy enhanced SVM and its application in predicting the second major. *Mathematical Problems in Engineering,* **2017**: 9316713. https://doi.org/10.1155/2017/9316713.

Widodo, D.S., Utama, D.M. (2019). Analisis model sustainable economic order quantity dengan mempertimbangkan emisi karbon dan batasan kapasitas gudang untuk menekan total biaya persediaan. *TEKNIK,* **40**(3): 169-175.

Yang, M.F., Tseng, W.-C. (2015). Deteriorating inventory model for chilled food. *Mathematical Problems in Engineering,* **2015**: 816876. doi: http://dx.doi.org/10.1155/2015/816876.

Yao, Y., Evers, P.T., and Dresner, M.E. (2007). Supply chain integration in vendor-managed inventory. *Decision Support Systems,* **43**(2): 663-674.

Zanoni, S., Zavanella, L. (2007). Single-vendor single-buyer with integrated transport-inventory system: Models and heuristics in the case of perishable goods. *Computers and Industrial Engineering,* **52**(1): 107-123.

Appendix

Appendix A

This section describes the assumptions, notations, and the mathematical model of the SVSB inventory model with exponential quality degradation for food products. The assumptions in this problem are 1) Quality of raw materials decreases exponentially during storage, 2) Production rate and demand rate are constant, 3) Delivery lead time is negligible, 4) Shortage and backorder are not allowed, 5) Production rate > demand rate, and 6). Therefore, the exponential quality degradation rate for the one cycle is 0 to 1.

The notations used in this model include:

P : production rate for producing the finished product (units/month)

D : number of demands for finished products (units/month)

λ : conversion coefficient of the finished product to raw material

q_r : the size of the raw material order (unit)

q_p : finished product delivery size (unit)

k : rate of quality degradation of raw materials (quality units/month)

Q_{max} : maximum quality of raw material (quality unit)

Q_{min} : minimum quality of raw materials (quality units)

$Q(t)$: remaining quality at time t for raw material (quality units)

c_{loss} : costs due to quality degradation of raw materials (IDR/quality units/month)

c_{sale} : cost of purchasing finished products from buyers to vendors (rupiahs/order)

c_r : the cost of purchasing raw materials (rupiahs/order)

c_p : costs for processing the finished product (IDR/unit)

A_r : transportation costs for the procurement of materials (IDR/order)

A_p : transportation costs for the delivery of the finished product (IDR/delivery)

S_p : installation costs for processing the finished product (IDR/month)

H_r : raw material storage costs (IDR/unit/month)

H_p : the cost of storing the finished product (IDR/unit/month)

I_{rm} : average raw material inventory at vendors (unit)

I_{pm} : the average finished product inventory at the vendor (unit)

I_{pr} : average finished product inventory at buyers (unit)

τ_{Δ} : finished product delivery intervals

τ_{max} : maximum duration for storage of raw materials (month)

τ_{sl} : the expiration time of the finished product (month)

τ_{Start} : the initial time of the deterioration of the finished product (month)

E_i : batch i product age when sent to the buyer (month)

R_i : total income in batch i (IDR/month)

P_{max} : maximum product price (IDR/unit)

p_{min} : minimum product price (IDR/unit)

$p(t)$: product price based on product age the $t-th$ (IDR/unit)

L : total costs due to quality degradation of raw materials (IDR/month)

TC_{rm} : the total cost of the raw material inventory system at the vendor (IDR/month)

TC_{pm} : the total cost of the finished product Inventory system at the vendor (IDR/month)

TC_{pr} : the total cost of the finished product Inventory system at the buyer (IDR/month)

JTR : Total income integrated Inventory system (IDR/month)

JTP : the total profit of the integrated Inventory system (IDR/month)

m : frequency of ordering raw materials (times/order)

T : time/length during the Inventory cycle (month)

n : delivery frequency of finished products to buyers (times/delivery)

The researcher proposes a mathematical model to solve the SVSB problem in this section. The proposed mathematical model was developed based on the model constructed by Fauza et al (Gusti Fauza et al., 2018). Three parts of the proposed mathematical model are; 1) a mathematical model for the quality degradation of raw materials, 2) a mathematical model for value losses arising from quality degradation of finished products, and 3) a mathematical model for the average inventory of raw materials and finished products.

Based on Figure 2, there were three types of models to determine the average inventory, namely raw materials at the vendor (I_{rm}), the finished product at the vendor (I_{pm}), and finished products in buyers (I_{pr}), which sequentially can be seen in the Equation (A-1), (A-2) and (A-3).

$$I_{rm} = \frac{\lambda D^2 T}{2m\lambda P} \tag{A-1}$$

$$I_{pm} = \frac{DT}{2n}\left(\frac{D}{P}(2-n) + (n-1)\right) \tag{A-2}$$

$$I_{pr} = \frac{DT}{2n} \tag{A-3}$$

In the quality degradation model, According to Rong, Akkerman, and Grunow (2011), the quality of food degradation depends on the environment and storage time. An illustration of the quality of food degradation is presented in Figure A-1. If *k = 0* is called a zero-order reaction, then the quality decreases linearly each time indicated by line A. However, if *k = 1* is denoted as the first-order reaction, then the quality decreases exponentially over time, shown by line B. Referring to Rong et al. (2011), the raw material which decreases exponentially is displayed using Equation (A-4).

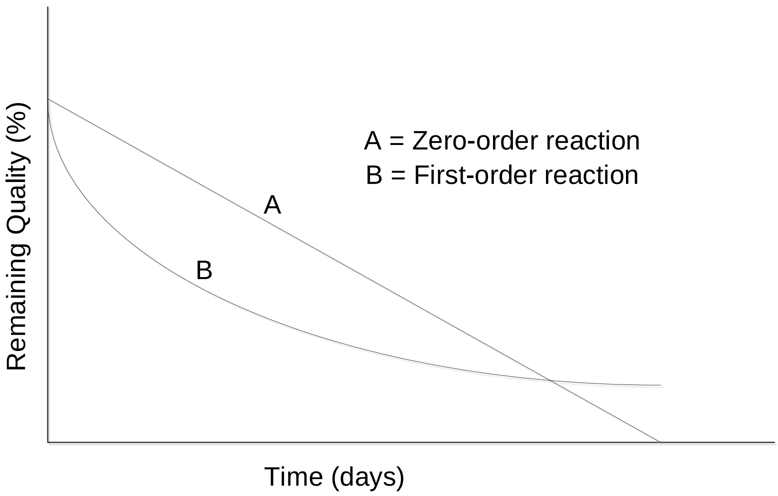

Figure A-1. Illustration of decreased quality of food products

$$Q(t) = Q_{max}\, e^{-kt} \tag{A-4}$$

Where $Q(t)$ is the level of quality remaining at the t-th time. The loss of quality in the period 0 to t was the Q_{max} to $Q(t)$, which can be denoted by $\Delta Q(t)$ so that it is stated by Equation (A-5).

$$\Delta Q(t) = Q_{max}(1 - e^{-kt}) \tag{A-5}$$

The total cost of quality loss per unit time $L(m,T)$ of raw materials for all *batches* during one production cycle can be represented by Equation (A-6).

$$L(m,T) = c_{loss}\frac{m\lambda P}{T}\int_0^{\frac{\lambda DT}{m\lambda P}} \Delta Q(t)\,dt \tag{A-6}$$

The total cost of the raw material inventory system at the vendor ($TC_{rm}(m,T)$)) is the sum of the costs of purchasing, transportation, storage, and quality degradation, presented in Equation (A-7).

$$TC_{rm}(m,T) = c_r\lambda D + A_r\frac{m}{T} + H_r\frac{\lambda D^2 T}{2m\lambda P} + c_{loss}\frac{m\lambda P}{T}\int_0^{\frac{\lambda DT}{m\lambda P}} \Delta Q(t)\,dt \tag{A-7}$$

The total cost of the finished product Inventory system at the vendor ($TC_{pm}(n,T)$) is the sum of processing costs, installation costs, and storage costs presented in Equation (A-8).

$$TC_{pm}(n,T) = c_p D + \frac{S_p}{T} + H_p\left(\frac{DT}{2n}\left(\frac{D}{P}(2-n) + (n-1)\right)\right) \tag{A-8}$$

The total cost of the finished product Inventory system at the buyer ($TC_{pr}(n,T)$) is the sum of the purchase costs, transportation costs, and storage costs presented in Equation (A-9).

$$TC_{pr}(n,T) = c_{sale}D + A_p\frac{n}{T} + H_p\left(\frac{DT}{2n}\right) \tag{A-9}$$

In the revenue model from buyers, based on Gusti Fauza et al. (2016), buyers set selling prices to consumers in three regions. The batch's age determines the price of each batch of product before it is shipped. Figure A-2 depicts the pricing structure based on the shelf life of each batch. The selling price before quality degradation occurs theτ_{Start} which was the maximum product price of p_{max} (region I). Furthermore, the remaining stock was sold at a discount price to attract more demands (region II). Meanwhile, products that have expired (reaching τ_{sl}) were set at the lowest price of p_{min} (region III). The price reduction policy is formulated using Equation (A-10).

$$p(t) = \begin{cases} p_{max} & 0 \le t < \tau_{Start} & region\ I \\ p_{min} + \frac{p_{max} - p_{min}}{\tau_{sl} - \tau_{start}}(\tau_{sl} - t) & \tau_{Start} \le t < \tau_{sl} & region\ II \\ p_{min} & t \ge \tau_{sl} & region\ III \end{cases} \tag{A-10}$$

The buyer accepted a *batch* that has E_i less than τ_{Start} to generate more income. The batch age (E_i) is denoted in Equation (A-11). Each i batch received by the buyer followed the 3 cases according to when the product was last consumed or $E_i + \tau_{\Delta}$.

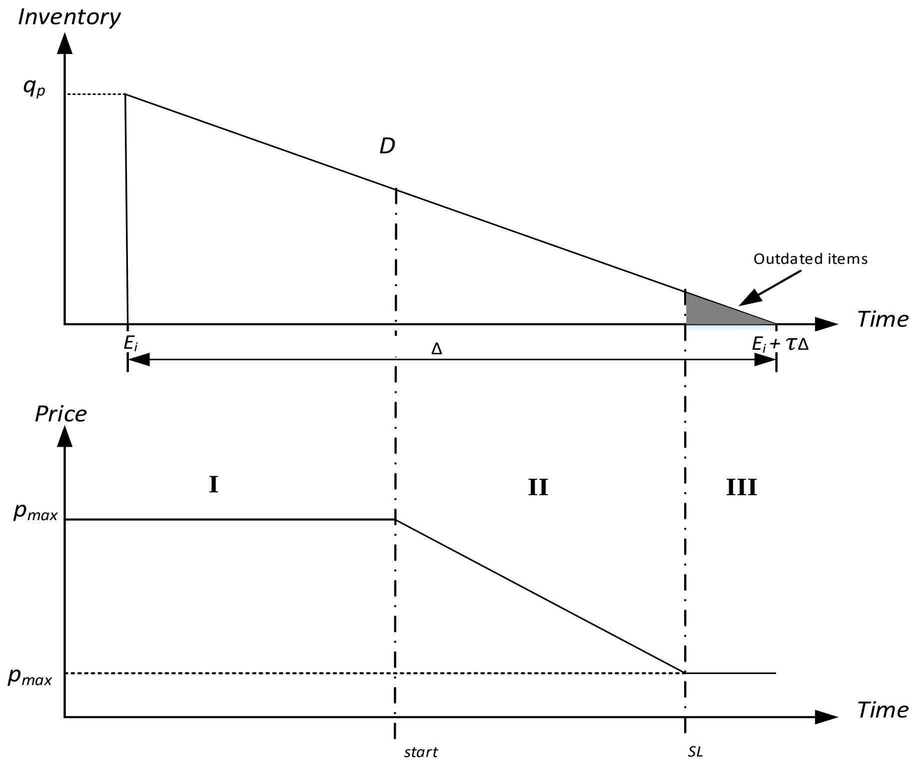

Figure A-2. Price function based on the shelf life of each batch

$$E_i = (i-1)\frac{T}{n} - (i-2)\frac{DT}{nP} \quad \text{(A-11)}$$

Case 1: $E_i + \tau_\Delta < \tau_{Start},$ the annual revenue earned from this batch is determined by Equation (A-12).

$$R_i(n,T) = \frac{D}{T} P_{max}\tau_\Delta \quad \text{(A-12)}$$

Case 2: $\tau_{Start} \leq (E_i + \tau_\Delta) < \tau_{sl}$, the annual revenue earned from this batch is determined by Equation (A-13).

$$R_i(n,T) = \frac{D}{T}\left[P_{max}(\tau_{Start} - E_i) + \int_{\tau_{Start}}^{E_i + \tau_\Delta} p(t)dt\right] \quad \text{(A-13)}$$

Case 3: $E_i + \tau_\Delta \geq \tau_{sl},$ Equation (A-14) represents the annual revenue function of this batch.

$$R_i(n,T) = \frac{D}{T}\left[P_{max}(\tau_{Start} - E_i) + \int_{\tau_{start}}^{\tau_{sl}} p(\mathrm{t})dt + p_{min}(E_i + \tau_\Delta - \tau_{sl})\right] \quad \text{(A-14)}$$

Total income integrated Inventory system ($JTR(T,n)$) is the sum of the vendor's income with the buyer's income presented in Equation (15).

$$JTR(T,n) = c_{sale}D + \sum_{i=1}^{n} R_i(T,n) \quad \text{(A-15)}$$

Then the total integrated inventory profit can be calculated by subtracting the total system cost of raw materials, finished products at vendors, and finished products at buyers (Equations A-7, A-8, and A-9) to total system revenue (Equation A-15). Finally, the total profit is formulated in Equation A-16.

This study implemented three decision variables the frequency of raw material orders (m), frequency of delivery of finished products to buyers (n), and time during the Inventory cycle (T). The decision variable for ordering frequency (m) and delivery *(n)* is an integer > 0. The time decision variable during the Inventory cycle (T) is a real number with a range of 0 to 1. The mixed-integer non-linear programming model for SVSB inventory model problems is described as follows:

Maximize :

$$JTP(m,T,n) = JTR(T,n) - \left(TC_{rm}(m,T) + TC_{pm}(n,T) + TC_{pr}(n,T)\right) \quad \text{(A-16)}$$

Subject to :

$P \geq D$; (A-17)

$E_i < \tau_{start}$; for *i* = 1, 2, ...*n* (A-18)

T > 0 ; (A-19)

m, n > 0 (integer number) ; (A-20)

Equation (A-16) is the objective function of the SVSB inventory model problem to maximize profit. The constraint of Equation (A-17) ensures that the production level can meet all demands. Equation constraint (A-18) provides that all batch *i* (E_i) arrive at the buyer's warehouse before the initial time of deterioration of the finished product. Equation constraints (A-19) and (A-20) guarantee that the decision variable is not zero and integer number.

Appendix B

Table B-1.
The effect of changes in m on cost and profit

m	n	T	k	λ	JTP (IDR)	$TCrm$ (IDR)	$TCpm$ (IDR)	$TCpr$ (IDR)	JTR (IDR)
1	4	0.76	0.50	7.2	57,047,690	78,045,069	868,974	110,658,265	246,620,000
5	4	0.76	0.50	7.2	65,141,311	69,951,448	868,974	110,658,265	246,620,000
10	4	0.76	0.50	7.2	65,966,234	69,126,526	868,974	110,658,265	246,620,000
15	4	0.76	0.50	7.2	66,027,751	69,065,008	868,974	110,658,265	246,620,000
20	4	0.76	0.50	7.2	65,895,146	69,197,613	868,974	110,658,265	246,620,000
25	4	0.76	0.50	7.2	65,684,361	69,408,398	868,974	110,658,265	246,620,000
30	4	0.76	0.50	7.2	65,434,338	69,658,421	868,974	110,658,265	246,620,000
35	4	0.76	0.50	7.2	65,161,838	69,930,921	868,974	110,658,265	246,620,000
40	4	0.76	0.50	7.2	64,875,266	70,217,493	868,974	110,658,265	246,620,000
45	4	0.76	0.50	7.2	64,579,301	70,513,458	868,974	110,658,265	246,620,000
50	4	0.76	0.50	7.2	64,276,755	70,816,004	868,974	110,658,265	246,620,000
55	4	0.76	0.50	7.2	63,969,419	71,123,341	868,974	110,658,265	246,620,000
60	4	0.76	0.50	7.2	63,658,487	71,434,272	868,974	110,658,265	246,620,000
65	4	0.76	0.50	7.2	63,344,789	71,747,970	868,974	110,658,265	246,620,000
70	4	0.76	0.50	7.2	63,028,916	72,063,844	868,974	110,658,265	246,620,000
75	4	0.76	0.50	7.2	62,711,302	72,381,458	868,974	110,658,265	246,620,000
80	4	0.76	0.50	7.2	62,392,273	72,700,486	868,974	110,658,265	246,620,000
85	4	0.76	0.50	7.2	62,072,079	73,020,680	868,974	110,658,265	246,620,000
90	4	0.76	0.50	7.2	61,750,914	73,341,845	868,974	110,658,265	246,620,000
95	4	0.76	0.50	7.2	61,428,931	73,663,829	868,974	110,658,265	246,620,000
100	4	0.76	0.50	7.2	61,106,252	73,986,508	868,974	110,658,265	246,620,000

Table B-2.
The effect of changes in n on cost and profit

m	$\boldsymbol{n}$	T	k	λ	JTP (IDR)	$TCrm$ (IDR)	$TCpm$ (IDR)	$TCpr$ (IDR)	JTR (IDR)
12	1	0.76	0.50	7.2	65,869,299	69,063,241	1,031,238	110,656,220	246,620,000
12	5	0.76	0.50	7.2	65,987,568	69,063,241	858,16	110,711,033	246,620,000
12	10	0.76	0.50	7.2	65,706,299	69,063,241	836,52	111,013,937	246,620,000
12	15	0.76	0.50	7.2	65,393,244	69,063,241	829,31	111,334,204	246,620,000
12	20	0.76	0.50	7.2	65,072,243	69,063,241	825,70	111,658,811	246,620,000
12	25	0.76	0.50	7.2	64,748,063	69,063,241	823,54	111,985,154	246,620,000
12	30	0.76	0.50	7.2	64,422,295	69,063,241	822,10	112,312,365	246,620,000
12	35	0.76	0.50	7.2	64,095,618	69,063,241	821,07	112,640,072	246,620,000
12	40	0.76	0.50	7.2	63,768,373	69,063,241	820,30	112,968,089	246,620,000
12	45	0.76	0.50	7.2	63,440,750	69,063,241	819,69	113,296,313	246,620,000
12	50	0.76	0.50	7.2	63,112,862	69,063,241	819,21	113,624,682	246,620,000
12	55	0.76	0.50	7.2	62,784,782	69,063,241	818,82	113,953,156	246,620,000
12	60	0.76	0.50	7.2	62,456,557	69,063,241	818,49	114,281,708	246,620,000
12	65	0.76	0.50	7.2	62,128,221	69,063,241	818,22	114,610,322	246,620,000
12	70	0.76	0.50	7.2	61,799,797	69,063,241	817,98	114,938,983	246,620,000
12	75	0.76	0.50	7.2	61,471,304	69,063,241	817,77	115,267,682	246,620,000
12	80	0.76	0.50	7.2	61,142,754	69,063,241	817,59	115,596,413	246,620,000
12	85	0.76	0.50	7.2	60,814,157	69,063,241	817,43	115,925,169	246,620,000
12	90	0.76	0.50	7.2	60,485,521	69,063,241	817,29	116,253,946	246,620,000
12	95	0.76	0.50	7.2	60,156,853	69,063,241	817,16	116,582,741	246,620,000
12	100	0.76	0.50	7.2	59,828,156	69,063,241	817,05	116,911,551	246,620,000

Table B-3.
The effect of change in n on cost and profit

m	n	$\boldsymbol{T}$	k	λ	JTP (IDR)	$TCrm$ (IDR)	$TCpm$ (IDR)	$TCpr$ (IDR)	JTR (IDR)
12	4	0.05	0.50	7.2	51,013,081	79,353,726	1,918,908	114,334,283	246,620,000
12	4	0.10	0.50	7.2	59,538,464	73,419,052	1,323,916	112,338,566	246,620,000
12	4	0.15	0.50	7.2	62,330,595	71,484,296	1,128,925	111,676,183	246,620,000
12	4	0.20	0.50	7.2	63,689,474	70,549,458	1,033,933	111,347,133	246,620,000
12	4	0.25	0.50	7.2	64,475,100	70,014,540	978,94	111,151,417	246,620,000
12	4	0.30	0.50	7.2	64,974,141	69,679,541	943,95	111,022,367	246,620,000
12	4	0.35	0.50	7.2	65,309,453	69,458,746	920,39	110,931,412	246,620,000
12	4	0.40	0.50	7.2	65,542,465	69,309,300	903,97	110,864,267	246,620,000
12	4	0.45	0.50	7.2	65,707,304	69,207,391	892,31	110,812,995	246,620,000
12	4	0.50	0.50	7.2	65,824,445	69,138,736	883,98	110,772,834	246,620,000
12	4	0.55	0.50	7.2	65,906,920	69,094,242	878,08	110,740,753	246,620,000
12	4	0.60	0.50	7.2	65,963,414	69,067,850	874,00	110,714,734	246,620,000
12	4	0.65	0.50	7.2	65,999,942	69,055,364	871,32	110,693,376	246,620,000
12	4	0.70	0.50	7.2	66,020,798	69,053,787	869,73	110,675,681	246,620,000
12	4	0.75	0.50	7.2	66,029,134	69,060,921	869,03	110,660,917	246,620,000
12	4	0.80	0.50	7.2	66,027,312	69,075,118	869,03	110,648,534	246,620,000
12	4	0.85	0.50	7.2	66,017,138	69,095,118	869,63	110,638,111	246,620,000
12	4	0.90	0.50	7.2	66,000,018	69,119,940	870,72	110,629,323	246,620,000
12	4	0.95	0.50	7.2	65,977,060	69,148,811	872,22	110,621,910	246,620,000
12	4	1.00	0.50	7.2	65,949,153	73,419,052	1,323,916	112,338,566	246,620,000

Table B-4.
Effect of change in k on cost and profit

m	n	T	$\boldsymbol{k}$	λ	JTP (IDR)	$TCrm$ (IDR)	$TCpm$ (IDR)	$TCpr$ (IDR)	JTR (IDR)
12	4	0.76	0.1	7.2	66,800,729	68,292,029	868,97	110,658,265	246,620,000
12	4	0.76	0.2	7.2	66,606,809	68,485,950	868,97	110,658,265	246,620,000
12	4	0.76	0.3	7.2	66,413,636	68,679,123	868,97	110,658,265	246,620,000
12	4	0.76	0.4	7.2	66,221,206	68,871,552	868,97	110,658,265	246,620,000
12	4	0.76	0.5	7.2	66,029,518	69,063,241	868,97	110,658,265	246,620,000
12	4	0.76	0.6	7.2	65,838,567	69,254,191	868,97	110,658,265	246,620,000
12	4	0.76	0.7	7.2	65,648,351	69,444,408	868,97	110,658,265	246,620,000
12	4	0.76	0.8	7.2	65,458,866	69,633,893	868,97	110,658,265	246,620,000
12	4	0.76	0.9	7.2	65,270,109	69,822,650	868,97	110,658,265	246,620,000

Table B-5.
Effect of changes in λ on cost and profit

m	n	T	k	$\boldsymbol{\lambda}$	JTP *(IDR)*	$TCrm$ *(IDR)*	$TCpm$ *(IDR)*	$TCpr$ *(IDR)*	JTR *(IDR)*
12	4	0.76	0.5	1	124,803,970	10,288,789	868,97	110,658,265	246,620,000
12	4	0.76	0.5	2	115,324,220	19,768,539	868,97	110,658,265	246,620,000
12	4	0.76	0.5	3	105,844,470	29,248,289	868,97	110,658,265	246,620,000
12	4	0.76	0.5	4	96,364,719	38,728,040	868,97	110,658,265	246,620,000
12	4	0.76	0.5	5	86,884,969	48,207,790	868,97	110,658,265	246,620,000
12	4	0.76	0.5	6	77,405,218	57,687,540	868,97	110,658,265	246,620,000
12	4	0.76	0.5	7	67,925,468	67,167,291	868,97	110,658,265	246,620,000
12	4	0.76	0.5	8	58,445,718	76,647,041	868,97	110,658,265	246,620,000
12	4	0.76	0.5	9	48,965,967	86,126,791	868,97	110,658,265	246,620,000
12	4	0.76	0.5	10	39,486,217	95,606,542	868,97	110,658,265	246,620,000
12	4	0.76	0.5	11	30,006,467	105,086,292	868,97	110,658,265	246,620,000
12	4	0.76	0.5	12	20,526,716	114,566,043	868,97	110,658,265	246,620,000
12	4	0.76	0.5	13	11,046,966	124,045,793	868,97	110,658,265	246,620,000
12	4	0.76	0.5	14	1,567,216	133,525,543	868,97	110,658,265	246,620,000