Pricing Perishables with Uncertain Demand, Substitutes, and Consumer Heterogeneity

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ABSTRACT

Within the marketing window for perishables such as food products, demand uncertainty is complicated by price sensitivity and propensity to postpone purchase that is heterogeneous across consumers. These features pose substantial challenges to retailers when pricing multiple products over time and across consumer segments. Getting the dynamic profile of prices right has implications for performance of vertical food chains ranging from revenues to food waste. This paper proposes an approach to dynamic pricing that is demonstrated to improve performance within this setting.

Keywords: Dynamic pricing, perishables, uncertainty, demand substitution, robust optimization, waste management

1   Introduction

Perishability of products follows from physical and functional obsolescence. While the former may predictable, the latter depends on uncertain demand as consumers switch to substitute products. Uncertainty faced by firms is further complicated by postponement of purchase by consumers. A key management control within this context is pricing. As demand and product characteristics are dynamic, pricing must also be recognized as a dynamic problem. Setting the wrong intertemporal price profile can result in inventory accumulation or stock-outs, as well as inefficient profiles of product deterioration, or ending stocks. Suboptimal pricing may leave ending stocks and waste disposal cost. In each case, these implications can imply high costs associated with uncertainty. Setting the wrong prices across a set of substitute products may result in unintended changes in demand as consumers switch products.

A further complication results from heterogeneity of consumers that compose the demand faced by the firm. We consider two types of heterogeneity. Consumers differ with respect to price sensitivity, see e.g. Narasimhan (1984), Levedahl (1988), Vilcassim et al. (1987), and Gerstner et al. (1994). Second, consumers may differ by the timing of their purchase within the product life. Postponement of purchase may be strategic based on anticipated price discounts or may reflect a willingness to trade-off quality for price. By setting price directly or indirectly using segment directed discounts or coupons, uniform posted prices can be differentiated across consumer segments. To manage these costs and resulting profitability of perishables, firms must establish a pricing policy that defines the intertemporal price profile, as well as prices across substitute products and heterogeneous consumers.

The stochastic properties demand for perishables are difficult to characterize. From this perspective, our approach is to treat demand as uncertain and, following Knight's suggestion, to distinguish this uncertainty from risk where stochastic properties of demand can be characterized by a particular distribution and optimal pricing derived from a risk neutral optimization problem. Further, our approach

presumes the retailer has market power. As argued in the applied I/O literature, spatial separation of food retailers often establishes spatial local pricing power. Even when competitive conditions may set price, mark-downs or discounts in either the primary market or secondary channels are typically used at the firm-level to establish price profiles that set prices below or at the market price. To the best of our knowledge, the question of how to determine the optimal intertemporal price profile for perishables with uncertain demand and product substitution has not been considered in past literature.

Consideration of the problem of pricing perishable food products has received very limited attention. Past literature has considered food products and offered a theory that retailers set price as a markup over farm price as reviewed by Wohlgenant (2001). Such pricing ignores product perishability as well as the dynamic profile of uncertainty of demand. Observed retail pricing suggests occasional promotional sales are often used at the end of predicted product life following a period of fixed price, Li et al. (2006). A key feature of food products is their perishability. Within the short marketing window that characterizes most perishable food and ag products, demand is typically highly stochastic, spatially specific, difficult to predict, and reflects substitution across products by heterogeneous consumers. This combination of features poses substantial challenges to retailers when pricing products over time, across products, over spatial dimensions, and across consumer segments. By ignoring perishability, stocks at the end of product life amount to food waste.

2 Dynamic pricing strategies

Consider a supplier that coordinates production of *J* perishable products that are marketable within a finite time horizon with dates $t = 0,1,...T$. Demand is heterogeneous and is characterized by *S* consumer segments with each segment noted as $s \in S_0 = \{1,2,...,S\}$ where the value of *s* is interpreted as indicating market segment such that as *s* increases, consumers have preferences for progressively higher quality that is reflected in higher price. We note that this specification is consistent with the existence of alternative market channels. We define demand for each market segment as $\delta_s^j(t) \in \{\delta_1^j(t), \delta_2^j(t), ..., \delta_S^j(t)\}$ for product *j* for $j = 1,...,J$.

We consider only the marketing problem and suppose initial stocks are pre-determined by prior production decisions. At any time *t* in the season, the supplier offers a *1 x J* vector of fixed supplies $q_s(t) = (q_s^1(t),...,q_s^J(t))$ to each segment *s* that represents remaining inventory given initial stocks of $q_s^j(0)$. Thus, operationally the firm is faced with sunk costs for an inventory that must be sold before the end of the season. Here, we suppose the control available to achieve this goal is the intertemporal price policy. The intertemporal price policy defines a sequence of prices over time within the product life, as well as across substitute products and market segments. That is, the firm chooses a price vector for each time $t = 0,1,...T$ incorporating a price $p_s^j(t)$ for each product *j* for $j = 1,...,J$ and for each market segment *s* for $s = 1,...,S$ to maximize revenue. This specification allows the firm to set price policy for each product that is jointly optimal across products given that other products may be cannibalized.

By definition, uncertainty describes conditions when a firm anticipates stochastic factors will affect its performance, however, the firm has limited knowledge of the mechanism generating the stochastic outcomes. In contrast, full knowledge of such mechanisms is presumed when a stochastic environment is described as posing risk. Given knowledge of characterizing moments of data generating mechanism, a natural approach to decision-making under risk is to suppose decision-makers have preferences over such moments and set controls to optimize a functional representation of those preferences. An example is risk neutral preferences that motivate expected profit maximization. In sharp contrast, under uncertainty, knowledge of the shape of the distribution of stochastic factors or its moments, are not assumed. Here, we propose use of robust optimization to set performance controls in uncertain decision environments. Robust optimization for a single control problem has been recently presented by Lan, et al. (2008) and Birbil, et al. (2009). Our specification considers robust pricing across a set of substitutable products where demand across a spectrum of heterogeneous customer segments is uncertain. Our approach builds on Soyster (1973) and Ben-Tal and Nemirovski (1999). The intuition of the robust optimization approach is simple. While the distribution of stochastic features of problem may be unknown, the decision maker may be able to define boundaries of variation. Using simulation across the bounded set of possible stochastic outcomes, the best choice can be made.

Case 1: Dynamic perishable pricing under risk and risk neutral preferences

We first consider the case of risk neutral dynamic pricing or equivalently where the firm forecasts demand based on a known distribution of stochastic demand, accepting those forecasts as its expectations. Where the firm's objective is linear in the stochastic factors, such an approach is equivalent to one where the firm is risk neutral, having preferences only for the first moment of a known distribution over the stochastic factor. Given fixed initial supply, the firm seeks a pricing strategy to maximize revenue and ensure zero ending stocks. Product substitutability implies that demand for a product *j* depends on the prices for a product *j* as well as those for other products in the product set, noted as a vector with subscript, $-j$, or where particular products are indexed with superscript *k*. We specify demand as follows:

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t)p_s^k(t), \quad \forall s, j, t$$

Our specification considers demand by segment *s* such that discrete choices by consumers across the product set define a continuous function in prices. Together, the specified system for J products can be viewed as a first-order Taylor series approximation of a complete demand system. We suppose demand is a negative monotonic function of the own price *j* and that of substitutes *–j*. Maglaras and Meissner (2006) or Perakis and Sood (2006) present similar specifications. We interpret the parameters $a_s^j(t) \in \Re_+^{S \times J \times (T+1)}$ as defining the market potential (i.e. maximum quantitative scale) for product *j* and segment *s* at time *t*. We view market potential as stochastic. The parameters $\beta_s^j(t) \in \Re_+^{S \times J \times (T+1)}$ and $\gamma_s^j(t) \in \Re_+^{S \times J \times (T+1)}$ represent price sensitivity of products *j* and *–j*, respectively. Note that our specification assumes consumers prefer to substitute products available in their segment rather than downgrading or upgrading to other segments. This is consistent with consumer loyalty found across particular market channels, stores, or branded versus private label products. We incorporate this specification by requiring that products are differentiated by market segment such that each product type *j* will be differentiated by market segment *s* such that its price will increase with *s*, see Birbil, et al. (2009) for a similar specification.

The firm's pricing problem is complicated by a mismatch between product availability and time of purchase within the product's life. That is, observation suggests consumers do not purchase instantaneously when initial supply is provided to the market. We consider this problem as a passive feature of demand, rather than a result of strategic behavior by consumers. This seems reasonable as product quality diminishes over product life. To differentiate quantity purchased from level of demand at any time, we define the proportion of demand that is postponed by customers each time period as $\eta_s^j(t) \in \Re_+^{S \times J \times (T+1)}$ and allow it to vary across segments and products. From the firm's perspective, the delayed purchase demand amounts to a shift of demand to future periods during the finite product life. We define the cumulative demand function given postponement at any time *t* as:

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))) = (1 - \eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))], \quad t = 1...T, \quad \forall s, j$$

Intuitively, this specification includes demand that is based on delayed demand postponed from last period (in the first expression in the squared-brackets) and that portion of current demand that will be postponed to next period (based on the second expression in the squared-brackets). In the absence of replenishment, the firm controls available inventory by setting price to maximize the season's revenue given stochastic demand across consumer segments and substitute products.

We first consider optimal pricing under a risk setting. Within this problem as summarized below, we suppose the firm chooses price policy to set a dynamic profile of prices over time, across substitute products, and across consumer segments to maximize expected revenue equation (1) based on an expectation of delayed demand. To conserve notation, we define expected market potential using a redefined notation, $a_s^j(t)$. We assume initial supply of the product is fixed and exogenous to the pricing problem. In the beginning, a fixed initial stock of each product is available, equation (6). At any time during the product life, expected demand, equation (4), is affected by postponement of purchase, equation (3), available supply is reduced each period, equation (2) and available supply must be positive or zero, equation (5). Thus, we do not constrain ending stock to be zero as that may not make economic sense. We constrain the price policy to ensure that prices are nonnegative, equation (7), that prices across

segments are ordered, equation (8), that prices are set such that demand will be nonnegative, equation (9), and we assume demand at zero prices will not reflect any delayed purchases, equation (10). The firm is assumed to be risk neutral and draws forecasts of demand intercepts as random parameters from known independent stochastic density functions. Given risk neutrality, we specify the risk neutral optimal pricing problem as one of expected revenue maximization:

$$R(p_s^j(t), p_s^{-j}(t)) = \max_{p_s^j(t), \forall j, s, t} \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} [p_s^j(t) \tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))] \tag{1}$$

s.t.

$$q_s^j(t) = q_s^j(t-1) - \tilde{\delta}_s^j(p_s^j(t), p_s^j(t))), \quad t = 1...T, \ \forall s, \forall j \tag{2}$$

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))) = (1 - \eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))], \ t = 1...T, \ \forall s, j \tag{3}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t), \ \forall s, j, t \tag{4}$$

$$q^j(t) \geq 0, \qquad \forall t, j \tag{5}$$

$$q^j(0) = q_0^j, \quad \forall j \tag{6}$$

$$p_s^j(t) \geq 0, \quad \forall t, j, s \tag{7}$$

$$p_s^j(t) \geq p_{s'}^j(t), \ s > s' \in S_0 \tag{8}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) \geq 0 \tag{9}$$

$$\tilde{\delta}_s^j(p_s^j(0), p_s^{-j}(0)) = \delta_s^j(p_s^j(0), p_s^{-j}(0)), \ \forall s, j \tag{10}$$

Case 2: Robust dynamic pricing for perishable under uncertainty

Define stochastic potential demand a_s^j for each product and consumer segment as having a best estimate $\bar{a}_s^j(t)$. Define the uncertainty set U_d for stochastic potential demand a_s^j as bounded by limits that are set by a scaling parameter $\theta_s^j(t)$ that scales our best estimate as in equation (11).

$$a_s^j(t) \in [\bar{a}_s^j(t)(1 - \theta_s^j(t)), \bar{a}_s^j(t)(1 + \theta_s^j(t))] \ \text{where} \ \bar{a}_s^j(t) \in \Re^{J \times S \times (T+1)}, \theta_s^j(t) \in \Re^{J \times S \times (T+1)}. \tag{11}$$

Thus, rather than supposing the agent has knowledge of the stochastic distribution of potential demand, in case 2 our specification supposes the manager holds a best estimate as well as a scaling parameter that defines limits on the set from which the manager presumes the stochastic potential demand is drawn. Alternative specifications such as ellipsoidal and polyhedral uncertainty sets are considered by Ben-Tal and Nemirovski (1999) and Bertsimas and Sim (2004). For a particular uncertainty set, suppose an optimal price policy is derived from the revenue maximizing problem describe above. It follows that the control problem has infinite number of constraints that describe possible uncertainty sets. Since a direct solution of such a problem is intractable, we manipulate the specification to transform the control problem to an equivalent, workable form. Specifically, we define a scalar *V* as the objective in (12) subject to the constraint in (13) which implies that at the optimum (13) is an equality. We propose the following deterministic problem as equivalent to the robust formulation with uncertain demand (derivation is available from authors):

$$R(p_s^j(t); p_s^{-j}(t)) = \max_{p_s^j(t), \forall j, s, t, V} V \tag{12}$$

s.t.

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t} (1 - \eta_s^j(\tau))(\eta_s^j(\tau - 1)) \right) \delta_s^j(0) \right. \right.$$
$$\left. \left. + \sum_{\tau=1}^{t} (1 - \eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(t)(1 - \theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \right\} \right] \geq V \tag{13}$$

$$\bar{a}_s^j(t)(1 - \theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \geq 0 \tag{14}$$

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i} (1 - \eta_s^j(\tau))(\eta_s^j(\tau - 1)) \right) \delta_s^j(0) \right.$$
$$\left. + \sum_{\tau=1}^{i} (1 - \eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(\tau)(1 + \theta_s^j(\tau)) - \beta_s^j(\tau) p_s^j(\tau) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(\tau) p_s^k(\tau) \right) \right) \geq 0, \forall s, j, t \tag{15}$$

$$p_s^j(t) \geq 0, \quad \forall t, j, s \tag{16}$$

$$p_s^j(t) \geq p_{s'}^j(t), \ s > s' \in S_0 \tag{17}$$

We note that (13) can be simplified to $\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} [p_s^j(t) \tilde{\delta}_s^j(p_s^j(t); t))] \geq V$ which parallels the objective

(1), however, defines *V* as a scalar value of the objective to be optimized.

3   Evaluation dynamic pricing policies under risk versus uncertainty

In this section, we compare our robust price policy against the price policy based on a forecast-based, risk neutral model. First, we present optimal price policies and inventories for each of the models derived from a numerical example. Next, we consider how the robust policy varies as the extent of uncertainty varies across a set of randomly generated scenarios.

In our numerical experiments, we assume that consumer segment 1 focuses on the highest quality and priced products. Parameters for numerical experiments are given in Table 1. We limit our consideration to demand parameters satisfying a particular segmentation of the consumer population based on ranking defined as:

Definition 1. Customer segment s is of a rank that is higher than that of segment s' if $a_s^j(t) \leq a_{s'}^j(t)$, $\beta_s^j(t) \leq \beta_{s'}^j(t)$, and $\gamma_s^j(t) \leq \gamma_{s'}^j(t)$ for $\forall j$, t.

We assume demand parameters satisfy Definition 1. This implies as *s* decreases, segment rank increases, and market potential and price sensitivity decreases. We specify parameters to allow a sharp focus on own-price sensitivity, so cross-price effects $\gamma_s^j(t)$ are specified as equal for the two products as are the postponement proportions $\eta_s^j(t)$. The first three rows of Table 1 define values for the demand parameters. Demand for product 1 is parameterized as having greater market scale with values for a_s^j that are greater than those used for product 2. Own price response β_s^j for product 1 is about 10% less than that for product 2. The products are parameterized as substitutes though not strong substitutes. 20% of demand is delayed by consumers each period as shown by η_s^j. Initially, we assume a low degree of uncertainty, using $\theta_s^j = 0.02$. We suppose initial stocks q^j are smaller for segment 1 consumers than

segment 2, and larger for product 1 than for product 2. The simulation experiments were implemented using MATLAB and GAMS on a machine with Windows XP OS, T2300 CPU, 1.66 GHz, and 1 GB RAM.

Table 1.
Parameters for simulation studies. Case: *J=2, S=2, T=10.*

Parameters	Product 1		Product 2	
	Segment 1	Segment 2	Segment 1	Segment 2
$a_s^j(t)$	60	120	30	100
$\beta_s^j(t)$	0.5	1.5	0.6	1.6
$\gamma_s^j(t)$	0.1	0.3	0.1	0.3
$\eta_s^j(t)$	0.2	0.2	0.2	0.2
$\theta_s^j(t)$	0.02	0.02	0.02	0.02
$q^j(0)$	100	400	80	300

The optimal price strategies and inventory policies are shown in Figure 1. In the figure, solid and dotted lines represent optimal dynamic pricing strategy based on risk neutrality and expected demand and on our robust strategy based on uncertain demand, respectively. Red and blue lines are the strategies for segment 1 and 2, respectively. In the top-two panels, the product price dynamic profiles are presented. In the lower two panels, the inventory profiles over time induced by the price policy are presented.
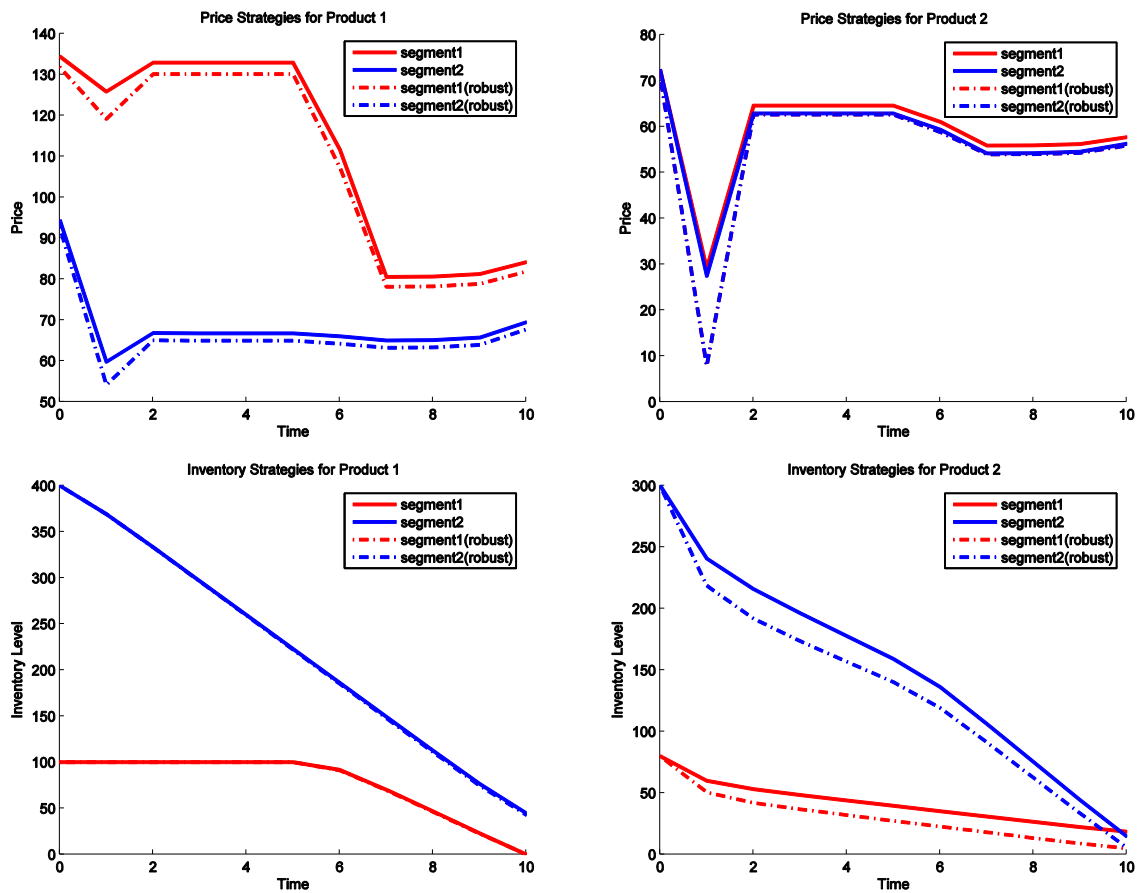
Figure 1. Dynamic price and inventory strategies

For each product, segment 1 prices are set above those set for segment 2. In both cases, a fixed price is not found to be optimal. Instead, initial price decreases are optimal, followed by a period of stable price. We view this initial price decrease as a means of clearing inventory given observed demand conditions no apparent in the initial time period. As shown in Figure 1, this price discounting increases sales and draws down inventory in the first period and prices are re-adjusted upward and maintained there until later periods when further discounting is set and maintained until inventory is cleared to an optimal level.

Under uncertainty, results show that optimal price strategy sets prices lower than does a price policy drawn from risk neutral optimization. This is consistent with aversion to future uncertainty that leads to pricing that induces greater sales early in the season to eliminate possible lost sales. As demand is uncertain, pricing is assumed to be adjusted to best clear inventories while optimizing the objective. Looking across market segments, we find the price for segment 1 is much higher than the price for segment 2 reflecting the economic benefits of differentiation. A similar price difference across segments is found under both demand conditions. For the inventories, we see that for the problem specified it is not optimal to set prices to clear inventories. We see optimal pricing results in some waste. We found that robust strategies for product 1 are nearly but not identical to those associated with the risk neutral case. The inventory trajectories for product 2 vary between the two market segments, however, in both segments inventories are drawn down faster under the uncertainty case with robust optimization. We consider the revenue outcomes next within a study of the implications of the level of uncertainty.

To consider sensitivity of firm revenue to the extent or level of uncertainty, we consider the implications of the value for $\theta_s^j(t)$. Intuitively, this parameter indicates the extent to which the bounds that define the uncertainty set differ from the best estimate of market potential, $a_s^{\ j}$. We simulate across the following set of values for $\theta_s^j(t)$: [2%, 5%, 7%, 10%, 15%]. This results in uncertainty defined by the limits of possible values increasing by 4%, 10%, 14%, 20% and 30%, respectively. Thus, for the uncertainty case, we derive five robust price strategy trajectories that can be compared to the risk neutral dynamic pricing

policy. Based on derived price trajectories, we generate random demand sets from a uniform distribution to generate demand realizations that define a set of scenarios. We draw our levels of market potential from the interval $[\overline{a}_s^j(t)(1-\theta_s^j(t)), \overline{a}_s^j(t)(1+\theta_s^j(t))]$. We derive 50 realizations (scenarios) for each setting. For each scenario, we derive corresponding revenue trajectories based on optimal dynamic price trajectories. To compare results, we note the risk neutral dynamic price trajectories (*D*) and robust trajectories (*R*) and, for each scenario, we report in Table 2 the optimal value (*ov*), as well as minimum (*min*), maximum (*max*), average (*ave*), and standard deviation (*sd*) of objective values.

Table 2.
Revenue characteristics under differing extents of uncertainty

	D_{ov}	R_{ov}	$D_{\max}$	$R_{\max}$	$D_{\min}$	$R_{\min}$	D_{ave}	R_{ave}	D_{sd}	R_{sd}
0.02	4.9712e+004	4.6854e+004	5.0784e+004	5.1757e+004	4.9178e+004	4.9876e+004	4.9861e+004	5.0743e+004	359.5613	331.9319
0.05	4.9712e+004	4.4984e+004	5.2824e+004	5.7034e+004	4.8641e+004	5.2732e+004	5.0463e+004	5.4621e+004	944.2957	910.4214
0.07	4.9712e+004	4.3783e+004	5.3800e+004	6.0095e+004	4.6941e+004	5.3378e+004	5.0089e+004	5.6313e+004	1.5416e+003	1.4640e+003
0.1	4.9712e+004	4.1946e+004	5.4003e+004	6.2711e+004	4.7689e+004	5.6926e+004	5.0485e+004	5.9412e+004	1.6425e+003	1.4787e+003
0.15	4.9712e+004	3.7643e+004	5.5573e+004	6.5213e+004	4.5801e+004	5.7144e+004	5.1140e+004	6.1633e+004	2.6568e+003	2.2816e+003

From the simulations of this experiment, robust optimization solutions are shown to be more stable (based on standard deviation) than risk neutral-based price policy, i.e. $R_{sd} < D_{sd}$ across each value of θ. This result supports the recommendation that to stabilize revenue the adoption of robust price strategies be pursued relative to forecast-based strategies. However, it is natural to ask what is the cost with respect to revenue achievable? Table 2 shows that $D_{ov} > R_{ov}$. However, we note that as revenue is stochastic, the optimum may not be achievable. Instead, we consider the average revenue which shows that $D_{ave} < R_{ave}$ at all levels of uncertainty. Thus, our robust pricing policy provides both an improved average outcome as well as reduced variation in revenue as we found $R_{sd} < D_{sd}$.

The dominance of our robust dynamic pricing is further supported by considering minimum, maximum revenues achievable. Further, Table 2 shows that the values of R_{ov} are decreasing in the amount of uncertainty and smaller than D_{ov}. Across each metric for each level of uncertainty, Table 2 shows our proposed robust dynamic pricing under uncertainty outperforms the risk neutral dynamic pricing that only considers risk.

4   Conclusions and further studies

In this paper, we have presented two dynamic pricing models: i) risk neutral optimization and ii) a robust optimization under uncertainty. Robust optimization allows the price policy to be derived such that is guarantees performance falls within specified boundaries given the degree of uncertainty. In other words, our robust models can prevent a loss associated with significant demand decreases that fall outside of such boundaries. In this way, relative to risk neutral pricing, our robust pricing is found to reduce ending inventories which are specified as waste in our model.

Our experiments for these dynamic pricing approaches show that the robust price strategies are very stable across different degrees of uncertainty and differing demand parameterizations. Even if a firm encounters a sudden and significant negative demand shock, a certain range of performance is guaranteed by the robust policy and this ensures that our robust approach dominates the risk neutral dynamic pricing policy.

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APPENDIX. Derivation of Robust Optimization model

By addressing uncertainty set (A.12) and manipulating objective function, we have

$$R(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t), \forall j, s, t} V \tag{A.1}$$

$$\text{s.t.} \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} [p_s^j(t) \tilde{\delta}_s^j(p_s^j(t); t))] \geq V \tag{A.2}$$

$$q_s^j(t) = q_s^j(t-1) - \tilde{\delta}_s^j(p_s^j(t), p_s^j(t))), \quad t = 1...T, \ \forall s, \forall j \tag{A.3}$$

$$\tilde{\delta}_s^j(p_s^j(t)) = (1 - \eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))], \ t = 1,...,T, \ \forall j \tag{A.4}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t)p_s^k(t), \ \forall s, j, t \tag{A.5}$$

$$q^j(t) \geq 0, \qquad \forall t, j \tag{A.6}$$

$$q^j(0) = q_0^j, \quad \forall j \tag{A.7}$$

$$p_s^j(t) \geq 0, \quad \forall t, j, s \tag{A.8}$$

$$p_s^j(t) \geq p_{s'}^j(t), \ s > s' \in S_0 \tag{A.9}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}) \geq 0 \tag{A.10}$$

$$\tilde{\delta}_s^j(p_s^j(0), p_s^{-j}(0)) = \delta_s^j(p_s^j(0), p_s^{-j}(0)), \ \forall s, j \tag{A.11}$$

$$\forall a_s^j \in [\bar{a}_s^j(1 - \theta_s^j), \bar{a}_s^j(1 + \theta_s^j)], \ \forall t, j, s \tag{A.12}$$

Note that the constraint (A.12) makes infinite number of constraints for the revenue maximizing problem. Thus, we need to manipulate this constraint.

Here, we know that constraint (A.5) over (A.12) can be reformulated as follows:

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t)p_s^k(t)$$

$$\geq \bar{a}_s^j(t)(1 - \theta_s^j(t)) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t)p_s^k(t) \geq 0$$

Also, manipulation of (A.11) and (A.4) gives us:

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t)) = (1 - \eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))]$$

$$= \left(\prod_{\tau=1}^{t} (1 - \eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{t} (1 - \eta_s^j(\tau))^{t-\tau} \delta_s^j(p_s^j(\tau), p_s^{-j}(\tau))$$

The constraint (A.6) and the above equation becomes

$$q_s^j(t) = q_s^j(t-1) - \left(\left(\prod_{\tau=1}^{t} (1 - \eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{t} (1 - \eta_s^j(\tau))^{t-\tau} \delta_s^j(p_s^j(\tau), p_s^{-j}(\tau)) \right)$$

$$t = 1,...T, \ \forall j \ .$$

From the (A.7), we have (A.6) over (A.12) as follows:

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{i}(1-\eta_s^j(\tau))^{t-\tau}\delta_s^j(p_s^j(\tau), p_s^{-j}(\tau)) \right)$$

$$\geq q^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \right.$$

$$+ \sum_{\tau=1}^{i}(1-\eta_s^j(\tau))^{t-\tau}\left(\overline{a}_s^j(\tau)(1+\theta_s^j(\tau)) - \beta_s^j(\tau)p_s^j(\tau) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(\tau)p_s^k(\tau) \right) \right) \geq 0$$

for $\forall t, j, s$.

Also, the constraint (A.2)

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\delta_s^j(p_s^j(\tau), p_s^{-j}(\tau)) \right\} \right] \geq V$$

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \right. \right.$$

$$+ \sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\left(\overline{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t)p_s^k(t) \right) \right\} \right] \geq V$$

Finally, we have (11)-(16).