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Economic Dynamics of the German Hog-Price Cycle

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ABSTRACT

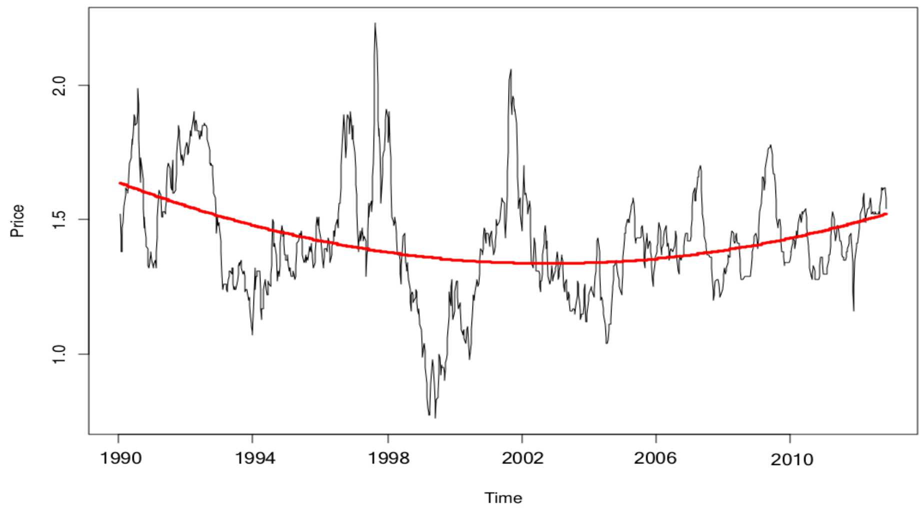
We investigated the economic dynamics of the German hog-price cycle with an innovative 'diagnostic' modeling approach. Hog-price cycles are conventionally modeled stochastically—most recently as randomly-shifting sinusoidal oscillations. Alternatively, we applied Nonlinear Time Series analysis to empirically reconstruct a deterministic, low-dimensional, and nonlinear attractor from observed hog prices. We next formulated a structural (explanatory) model of the pork industry to synthesize the empirical hog-price attractor. Model simulations demonstrate that low price-elasticity of demand contributes to aperiodic price cycling – a well know result – and further reveal two other important driving factors: investment irreversibility (caused by high specificity of technology), and liquidity-driven investment behavior of German farmers.

Keywords: Hog cycle; nonlinear dynamics; chaos; phase space reconstruction

1 Introduction

A time series plot of weekly German hog prices* exhibits persistent, aperiodic (non-repeating), and random-appearing oscillations (Figure 1). Agricultural economists conventionally theorize that such price volatility is due to cyclical adjustments made by stable markets to re-establish equilibrium after exogenous random shocks (Belair and Mackey, 1989; Mackey, 1988; Newbery and Stiglitz, 1981). Early work on the German hog-price cycle contended that the adjustments are cyclical due to naïve producer behavior characterized by linear cobweb price adjustments (Buchholz, 1982; Ezekiel, 1938; Hanau, 1928; Harlow, 1960; Waugh, 1964) and reasoned that cycling could not be a "permanent fixture of the pork industry" because countercyclical producer response would eventually eliminate it (Hayes and Schmitz, 1987, p. 762). Recent work asserts that producers are unable to predict future prices due to stochastic influences, and consequently treats the German hog-price cycle as a purely random phenomenon modeled as a randomly-shifting sinusoidal oscillation with time varying amplitudes (Parker and Shonkwiler, 2014).

* This is a weekly record of average producer prices for slaughtered pigs of quality E to P, in € per kg carcass weight, for the state of North Rhine-Westphalia, Germany from January 1990 to December 2011 (1144 observations). Source: Landesamt für Natur, Umwelt und Verbraucherschutz NRW.



Source: Landesamt für Natur, Umwelt und Verbraucherschutz NRW

Figure 1. Time series plot of hog prices and estimated trend

The conventional theory of price volatility is mathematically convenient for simulating irregular cycling. Stable markets can be modeled with linear equations of motion. The limitation of linear dynamics — that they can at most generate regular periodic cycling (i.e., 'limit cycles') (Kantz and Schreiber, 1997) — is overcome by adding an random error term that exogenously shifts a periodic cycle through time to simulate aperiodic cycling characterizing observed data. The downside is that economic theory is tethered to a stable-market hypothesis relying on random chance to explain price volatility as a transitory phenomenon. This falls woefully short of explaining real-world market dynamics if price volatility is instead persistent behavior due to systematic endogenous market instability.

Breakthroughs in nonlinear dynamics demonstrate that irregular and complex dynamic behavior can emerge from simple deterministic nonlinear interactions of system variables (Kantz and Schreiber, 1997; Kaplan and Glass, 1995). Agricultural economists have recognized the implication that nonlinear market models may be able to generate price volatility endogenously (Chavas and Holt, 1991, 1993; Holt and Craig, 2006; Holzer and Precht, 1993; Huffaker, 2010; McCullough et al., 2012; Streips, 1995). On the modeling side, Chavas and Holt (1993) demonstrated that a nonlinear market model of the US dairy industry could produce aperiodic price dynamics endogenously given highly-inelastic demand. Holt and Craig (2006) employed regime switching models to provide evidence of nonlinearity, regime dependent behavior, and structural change in the US hog-corn cycle over an almost 100-year study period. On the empirical side, several studies have tested observed hog-price data positive for nonlinear dynamics. Statistical tests by Chavas and Holt (1991) (using quarterly US data), and Holzer and Precht (1993) (using weekly German data), failed to reject the hypothesis of nonlinear price dynamics. Streips (1995) verified the results in Chavas and Holt (1991) for monthly data. McCullough *et al.* (2012) detected nonlinear dynamic structure in US livestock cycles.

We propose an innovative 'diagnostic' modeling approach that first empirically diagnoses real-world market dynamics embedded in observed prices, and then applies the diagnosis to inform the specification of theoretical models used to simulate and explain these dynamics. A diagnostic approach allows the data to 'speak' regarding whether a linear-stochastic or nonlinear-dynamic market model is most warranted in particular circumstances. Pre-modeling data diagnostics are useful because neither specification is a theoretical imperative *a priori*. Neither specification can be logically verified as 'the' accurate representation of reality because the truth of propositions can be ascertained only in closed systems, whereas a model represents an open real-world system in constant flux (Oreskes et al., 1994; Rykiel, 1996). Nor can either specification be logically verified by demonstrating a good fit between model output and observed data because very different models can be parameterized to provide a good fit (Hornberger and Spear, 1981). Oreskes *et al.* (1994) conclude that "…modelers should demonstrate the degree of correspondence between the model and the material world it seeks to represent..." (p. 644). Diagnostic modeling is a means of demonstrating correspondence.

We conduct data diagnostics with *Nonlinear Time Series Analysis* (*NLTS*) (Kantz and Schreiber, 1997; Schreiber, 1999) — an emerging empirical arm of nonlinear dynamics. *NLTS* comprises a battery of procedures to diagnose nonlinear system dynamics from a single observed time series. This is possible because any single variable records interactions with other system variables. As Farmer (1987) explained: "…the evolution of [a variable] must be influenced by whatever other variables it's interacting with. Their values must somehow be contained in the history of that thing. Somehow their mark must be there." (Gleick, 1987, p. 266). Famous naturalist John Muir intuited this result in the early nineteenth century observing: "When we try to pick something up by itself, we find it hitched to everything else in the universe" (Muir, 1911).

Failure to diagnose nonlinear-dynamic structure embedded in an observed price series provides evidence for adequacy of a linear-stochastic market model. Alternatively, a price series diagnosed positive for embedded nonlinear-dynamic structure provides evidence that observed volatility may be explained with a nonlinear market model. We can experiment with simple nonlinear-feedback structures (Larsen et al., 2014), and reasonably expect to find parsimonious specifications generating market dynamics corresponding to real-world complexity. These models can be used to explore factors that may be responsible for empirically-diagnosed market dynamics.

We apply diagnostic modeling to detect nonlinear market dynamics embedded in German hog prices, and to formulate a nonlinear market model simulating and explaining these dynamics.

2 The German hog-price cycle

The graph of German hog prices (Figure 1) shows that the average weekly price level decreases during the first decade and increases slightly thereafter. This trend is mainly caused by the change of the Common Agricultural Policy (CAP) of the European Union, starting with the McSharry reform in 1992. The CAP reform liberalized commodity markets by reducing the price support (i.e. intervention prices). This led to a strong decrease of grain prices during the nineties and the early years of the new century. Consequently, hog prices followed declining feed prices leaving the farmers' margins largely unchanged. The increase in average hog price in the latter part of the record is observed for most agricultural commodities.

Past studies of the US hog cycle normally rely on the hog to corn price ratio (Holt and Craig, 2006). This implies that the decision makers value the slaughter pigs in quantities of corn. This might have been a valid assumption in the past, but it is highly questionable for the present circumstances, at least under European conditions. With today's commonly used technology, significantly more than half of the total cost are fixed cost associated with the provision of the durable assets. For the past two decades, we found hardly any hog-to-feed price ratio for which the preferable choice would have been to leave capacities idle. Thus, short term production decisions are primarily driven by past investments, and are largely independent from current feed prices. Furthermore, farmers as well as feed suppliers can choose between different components. Consequently, the volatility of feeding cost will always be less than the volatility of a single feedstuff. Finally, changes of feedstuff prices will be encoded in the hog prices as far as there is a causality. Our empirical results to follow provide evidence for such causality.

For these reasons, we contend that representing the hog cycle by a hog-to-feed price ratio biases the analysis by mixing two phenomena with totally different origins: the hog price cycle on one hand and the volatility of barley prices on the other.[†] Increased barley price volatility is a recent phenomenon due to CAP reforms, whereas the hog cycle has existed for a long time caused by factors requiring further analysis. We focus our analysis on slaughter hog prices. Given nonlinear dynamic industry structure, slaughter-hog-price dynamics encode patterns of feedstuff prices.

[†] For example, Parker and Shonkwiler (2013) conclude that – contrary to the USA – the hog cycle in Germany is becoming more volatile. This however contradicts the pattern of slaughter pig prices that exhibits a slightly decreasing volatility in the most recent years (Figure 1).

3 Diagnostic modeling approach

We propose the diagnostic modeling approach depicted in Figure 2. Initially, *NLTS* is applied to empirically diagnose whether nonlinear system dynamics are embedded in an observed time series (leftward column). Subsequently, diagnosed nonlinear dynamics—along with knowledge about the industry structure and important characteristics of the technology—are applied to develop an explanatory model whose simulated dynamics can be validated for correspondence to empirically-diagnosed dynamics (rightward column).

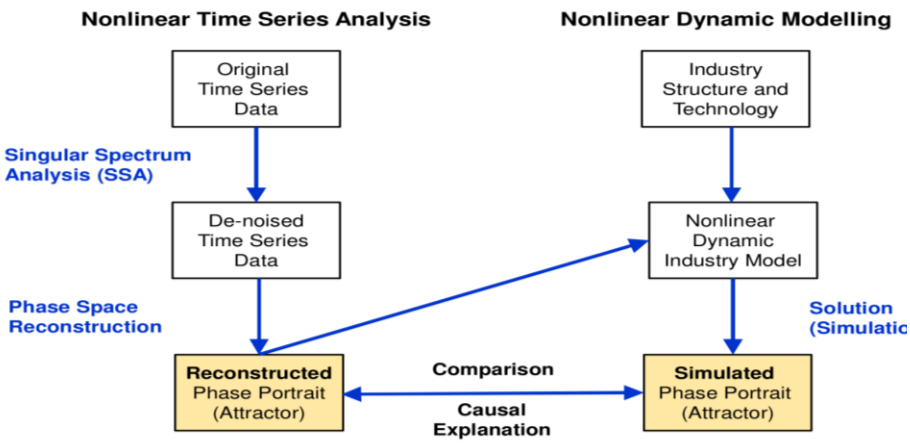


Figure 2. Diagnostic modeling approach

3.1 Singular Spectrum Analysis

The effectiveness of any empirical tool is limited by the quality of available data, and economic data are notoriously short and noisy. Time series length is important because, similar to other time series methods, *NLTS* requires stationary data. Stationarity requires that the "duration of the measurement is long compared to the time scales of the systems." (Schreiber, 1999, p. 33) Time series that are too short fail to provide an adequate sampling of important oscillatory patterns occurring at lower frequencies, so that these are grouped into linear or nonlinear trends. Conventional *NLTS* practice applies signal processing methods that detrend data to isolate detected oscillations (Greco et al., 2011).

Noisy time series data increase the difficulty of diagnosing embedded dynamic structure (Kot, 1988). Some noise may be due to observation or measurement errors most effectively treated as white noise that can be eliminated with linear filters. However, substantial variability in observed data—commonly attributed to random noise—may instead be due to complex deterministic dynamics resulting from nonlinear feedback interactions among system components.

Singular Spectrum Analysis (*SSA*) is a data-adaptive signal processing approach used to prepare data for *NLTS* by detrending data and separating signal (S) from noise (N) without losing dynamic structure (Elsner and Tsonsis, 2010; Ghil et al., 2002; Golyandina et al., 2001; Vautard, 1999). Initially, *SSA* embeds the price series, *P(t)*, into a 'trajectory matrix', *X,* whose columns are $K = N - L + 1$ single-period lagged vectors of *P(t)*, *N* is record length, and *L* is 'window length' restricted by $2 \leq L \leq N/2$. Window length is the only parameter that must be set to run *SSA*, and is conventionally selected proportional to the dominant spectral peak in the Fourier spectrum (Hassani, 2007).

A 'singular value decomposition' decomposes trajectory matrix *X* into the sum of 'empirical orthogonal functions'

(*EOF*), $X = \sum_{i=1}^{r} EOF_i$,

where $EOF_i = \sqrt{\lambda_i} EV_i PC_i^T$, *r = rank X*, and eigenvalues λ_i, eigenvectors $\left(EV_i\right)$, and principal components $\left(PC_i\right)$ are drawn from the eigensystem of the covariance matrix, XX^T. The sum of all eigenvalues measures the total variance in the time series (Ghil *et al*., 2002), and each eigenvalue measures the partial variance explained by their respective *EOFs*.

Next, the *EOFs* are arranged in rank order according to magnitude of their respective singular values, $\left(\sqrt{\lambda_i}\right)$, and then grouped to form the basis for trend, oscillatory, and unstructured-noise components. The initial *EOF* typically forms the basis for the trend component. Subsequent consecutive *EOF* pairs—whose eigenvectors oscillate with identical frequency in phase quadrature—are grouped to form the basis of oscillations. The trend and oscillatory components comprise the signal (S), and remaining EOF's constitute unstructured noise (N). The detrended signal omits the trend component, and thus includes only oscillatory components. An informative measure of signal strength is the sum of the eigenvalues associated with the trend and oscillatory components.

In the final step, 'diagonal averaging' of grouped *EOF* matrices converts them to vector time series of corresponding trend, oscillatory, and noise components (Golyandina et al., 2001).

3.2 Phase Space Reconstruction

NLTS applies *Phase Space Reconstruction* (Kantz and Schreiber, 1997; Schreiber, 1999) to determine whether nonlinear market dynamics can be reconstructed from a strong price signal, i.e., one that explains a substantial portion of total variability in observed prices.

Dynamic systems are composed of interrelated variables, and each point in phase space records the level of these variables in a given time period (the 'state' of the system). A unique trajectory passing through each point in phase space shows how system variables co-evolve. In low-dimensional nonlinear dynamic systems, system variables co-evolve from given initial conditions toward an 'attractor'—a geometric structure with "noticeable regularity" (Brown, 1996, p. 55). Examples include stable fixed points, stable limit cycles, and strange attractors upon which solution trajectories oscillate irregularly (Glendinning, 1994; Strogatz, 1994). Key topological properties of attractors include the 'correlation dimension' and the 'Lyapunov exponent' (Kaplan and Glass, 1995; Schreiber, 1999). The correlation dimension measures the attractor's geometric dimension, and thus indicates the minimum number of variables required to model real-world phase space. The Lyapunov exponent measures the average rate at which initially close points on the attractor exponentially diverge or converge, and thus indicates sensitivity to initial conditions. ‡ Strange attractors are characterized by low-dimensional fractal correlation dimensions and positive Lyapunov exponents.

We illustrate the concept of phase space with a simple linear market model in which producers myopically expect the price observed in the previous period:

‡ R-package 'tseriesChaos' was used for computing the correlation dimension, and Lyapunov exponent.

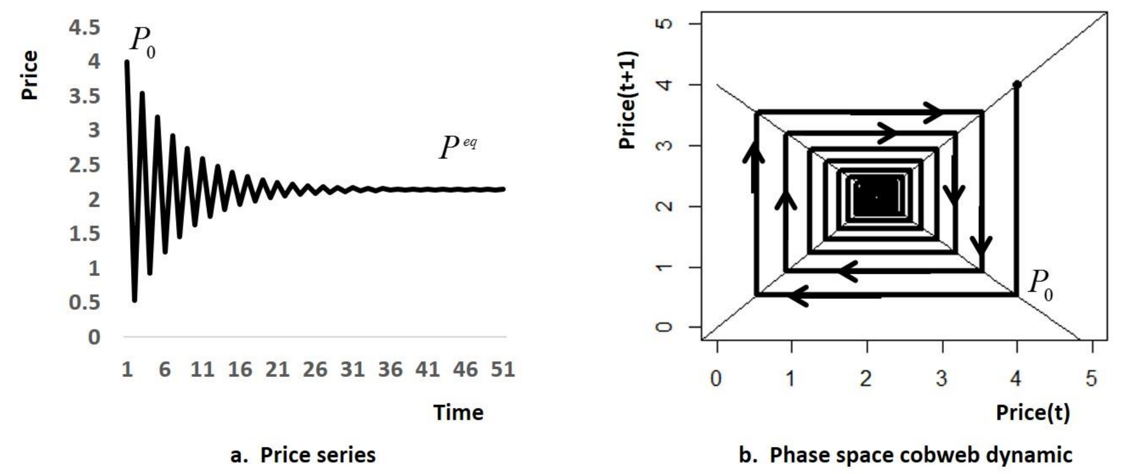


Figure 3. Linear cobweb price adjustment dynamics

Demand: $q_t^d = a - bp_t \quad (b > 0)$

Supply: $q_t^s = c + dp_{t-1} \quad (d > 0)$

Price dynamics: $p_t = \dfrac{a-c}{b} - \dfrac{d}{b} p_{t-1}$

Imposing a market equilibrium in which price equates demand with supply results in the price dynamics equation. Setting *a* = 20, *b* = 4.5, *c* = 2, *d* = 3.9, and initial price $p_0 = 4$ generates the market-clearing price series in Figure 3a. Prices solving the price dynamics equation clearly evolve along a dampened cycle from initial level p_0 toward equilibrium level p^{eq}. Phase space for this univariate price model re-plots the solution in p_t, p_{t+1} space, which highlights the transition of price from one period to the next (Figure 3b). Prices evolve cyclically toward a fixed-point attractor p^{eq} characteristic of the familiar stable cobweb dynamic.

In this simple example, we were able to solve for the market dynamic depicted in Figure 3b because we knew model equations. In reality, we don't know 'the' equations for real-world systems generating observed data—otherwise there would be no need for abstract models. Fortunately, Takens (1980) proved that a 'shadow' version of an attractor can be reconstructed from a single system variable without knowing system equations. This powerful result provides a sound mathematical foundation for *Phase Space Reconstruction* (Kot, 1988; Schreiber, 1999). Moreover, in this simple example, the price dynamic is obvious in price series—extensive time series analysis is not needed. The utility of *Phase Space Reconstruction* is to diagnose dynamic structure concealed in irregular and random-appearing data.

The 'time-delay' embedding method of *Phase Space Reconstruction* (Takens, 1980) represents the multidimensionality of real-world market systems by segmenting the detrended and filtered observed price series, $P_f(t)$, into a sequence of delay coordinate vectors:

$$P_f(t-d), P_f(t-2d), ..., P_f(t-(m-1)d),$$

where *d* is the 'embedding delay' and *m* is the 'embedding dimension' (i.e., the number of delayed coordinate vectors). The embedding delay is conventionally selected as the delay for which the mutual information function reaches its first minimum (Williams, 1997). The embedding dimension is conventionally selected with the 'false nearest neighbors' test. The selected dimension is that for which the percentage of 'false nearest neighbors' falls below a prescribed tolerance (Williams, 1997). [§]

[§] R-package 'tseriesChaos' was used for computing embedding delay and the embedding dimension.

If $m \geq 2n+1$, the reconstructed shadow attractor is guaranteed to share key topological properties with a reconstruction in any coordinate system, where *n* is the dimension of the real-world attractor (Takens, 1980). Since *n* is unobserved in practice, $m \geq n$ is generally considered adequate to reconstruct true system dynamics (Small and Tse, 2002). The scatterplot of the delay coordinate vectors depicts a trajectory in reconstructed shadow phase space representing a sampling or 'skeleton' of the real-world attractor (Ghil et al., 2002; Vautard, 1999).

.3 Surrogate Data Analysis

Surrogate Data Analysis is conventionally done to test whether apparent structure detected in an empirically reconstructed shadow attractor is more likely the figment of a mimicking stochastic process. A shadow attractor's topological properties are compared statistically with those taken from phase space reconstructed from randomized surrogate vectors (Small and Tse, 2002, 2003; Theiler et al., 1992).

Surrogate vectors are designed to destroy intertemporal patterns in the SSA-filtered record while preserving various statistical properties. Two conventional types of surrogate vectors are: *AAFT* (amplitude-adjusted Fourier transform) surrogates and *PPS* (pseudo phase space) surrogates. *AAFT* surrogates are generated as static monotonic nonlinear transformations of linearly filtered noise. They preserve both the probability distribution and power spectrum of the SSA-filtered data (Theiler et al., 1992). *PPS* surrogates test for the presence of a noisy limit cycle by preserving periodic trends in the SSA-filtered data while destroying chaotic structures (Small and Tse, 2003).[**]

Surrogate data testing proceeds by measuring topological properties associated with the phase space reconstructed from each surrogate vector. The mean from the distribution of each measure for the set of surrogate vectors is tested for significant difference from the corresponding empirical measure. Statistically insignificant differences indicate that detected empirical structure is more likely attributed to stochastic behavior.

We formulated a two-tailed test rejecting the null hypothesis of insignificant difference when mean surrogate topological properties are significantly above or below their empirical counterparts. Rejection occurs for the set of critical significance levels α_c satisfying:

$$\alpha_c \geq 2\left(1-\Phi\left|t\right|\right)$$

where the right-hand side of the inequality is the *p*-value for a two-tailed test (Minitab), $\Phi\left|t\right|$ is the CDF for the *t*-statistic with *N-1* degrees of freedom, and $\left|\ \right|$ is absolute value. Rejection of the null hypothesis indicates that the structure detected in the shadow attractor is not due to mimicking random behavior.

3.3 Diagnostic Modeling

NLTS-diagnosed dynamics can guide the specification of mechanistic models explaining real-world behavior, and also provide a specification test for how well simulated phase space corresponds to empirically-reconstructed phase space. An informative test for model specification is whether an empirically-reconstructed shadow attractor is similar to a simulated model attractor (Huffaker, 2010; Huffaker et al., 2003; Kot, 1988). In one application, Kot *et al.* (1988) reconstructed shadow attractors from observed time series on Copenhagen measles and chickenpox epidemics. Simulated attractors using the popular SEIR epidemiological model did not match shadow attractors until the SEIR model was modified to seasonally force the 'infection' parameter. Kot et al. (1988) concluded:

> "The agreement between the attractors of the models and those of the data are rather striking and suggest that simple deterministic models can capture the behavior…of at least some complex biological systems" (p. 92).

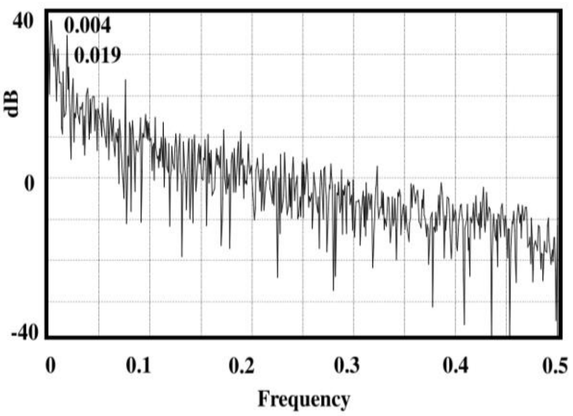
In another application, Huffaker *et al.* (2003) applied a generalized-logistic specification of student, faculty and administrator interactions in a university 'ecosystem' to simulate an attractor matching a stable-focus shadow attractor reconstructed from historic populations.

[**] We follow methods outlined in Kaplan and Glass (1995) and Small and Tse (2002) to write R-code generating AAFT and PPS surrogate vectors, respectively.

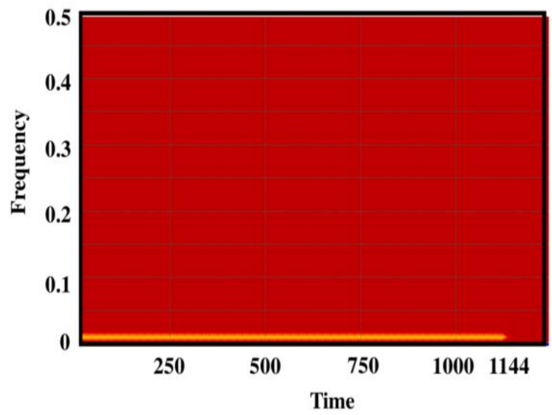
4 *NLTS* diagnostics of German hog-prices

We applied the *NLTS* framework (Figure 2) to diagnose possible low-dimensional nonlinear dynamic structure embedded in the German hog-price record.

Fourier Spectrum Analysis identifies dominant peak frequencies at 0.004 Hz (a 260-week or 5-year oscillation period) and 0.019 Hz (a 52-week or annual oscillation period) (Figure 4a). *Continuous Wavelet Analysis* verifies stationary power at the low frequency 5-year oscillation as required by subsequent analysis (Figure 4b).[††]



a. The Fourier frequency spectrum



b. Continuous wavelet time-frequency spectrum

Figure 4. Spectral analysis

Accordingly, the window length for *SSA* S/N separation was set at L = 520, which allows for 10 repetitions of the annual (52 month) oscillation period. The eigenvectors associated with *EOF* pairs 2,3 and 6,7 exhibit the 5-year and annual oscillations detected by the *Fourier* spectrum, respectively (Figure 5a,b). The isolated trend component[‡‡] and the composite *SSA*-reconstruction filtered of the unstructured-residual component are graphed against the observed hog-price record in Figure 5c. Compelling evidence for the strength of the signal in the SSA-reconstruction is that it accounts for 99% of the variation in the observed hog-price record.

We reconstructed a low-dimensional nonlinear shadow attractor from the detrended signal separated from the German hog-price record (Figure 6). The attractor has an embedding delay d = 20 weeks and an embedding dimension m = 3. It is a 'torus-type' attractor composed of nonrepeating 5-year and annual oscillations. The top view of the shadow hog-price attractor is shown in Figure 6b. The sampled trajectory makes four full 5-year revolutions around the attractor depicted in Figs. 6c-f. The attractor has a computed correlation dimension of 3.0, which indicates that a minimum of three variables are required to model the shadow hog-price attractor (Table 1). The computed Lyapunov exponent is 0.02, which indicates sensitivity to initial conditions (Table 1). Both measures are consistent with the existence of a strange German hog-price attractor.

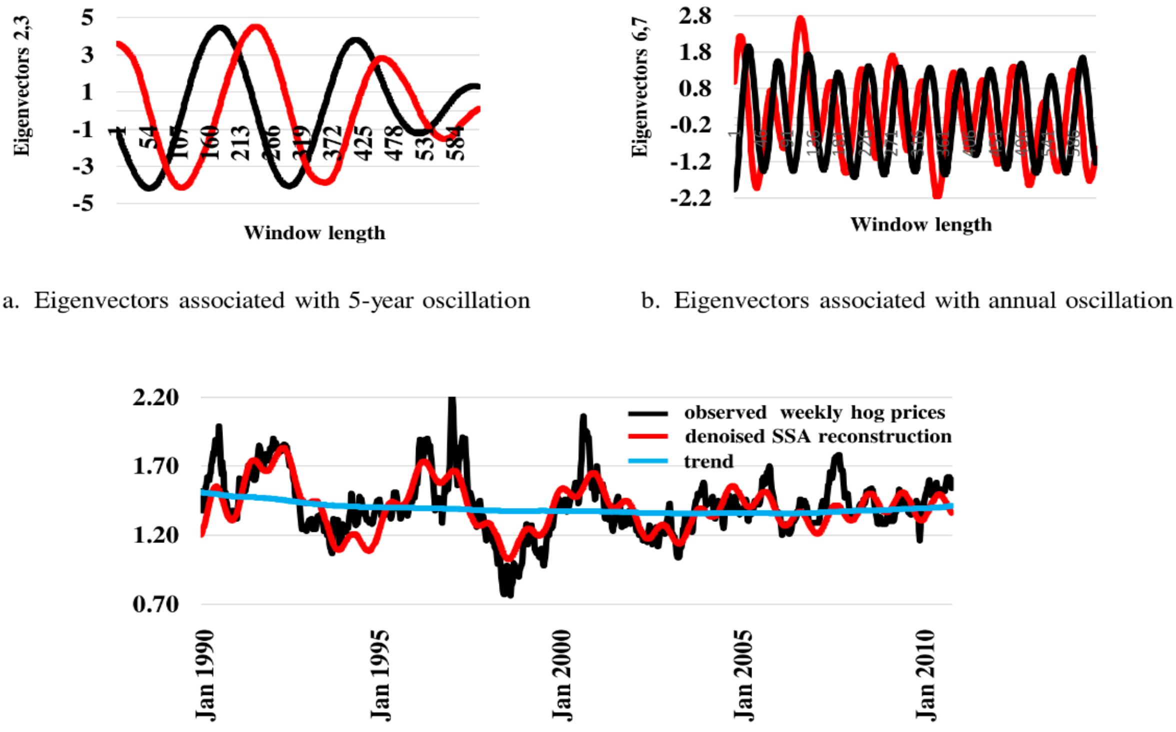
We tested the reconstructed shadow attractor against 100 *PPS* surrogate price vectors generated from the German hog-price series. Surrogate data tests soundly reject the null hypothesis that the nonrepeating cycling characterizing the shadow attractor is randomly generated with computed p-values effectively zero (Table 1).

[††] AutoSignal 1.7 (© SeaSolve Software Inc., 1999-2003) was used for Fourier spectral analysis and Continuous Wavelet Analysis.

[‡‡] During the period under consideration, there were no abrupt technological or structural changes that would have caused structural breaks. Thus, the detrended price series can be viewed as being generated under a relatively constant economic environment.

Table 1.
Results of Surrogate Data Tests

	Empirical Attractor	Surrogate Mean	Surrogate St Deviation	p-value
Correlation Dimension	3.06	3.37	0.22	0.00
Lyapunov Exponent	0.028	1.32	0.20	0.00



a. Eigenvectors associated with 5-year oscillation

b. Eigenvectors associated with annual oscillation

c. Reconstructed SSA components

Figure 5. Singular Spectrum Analysis

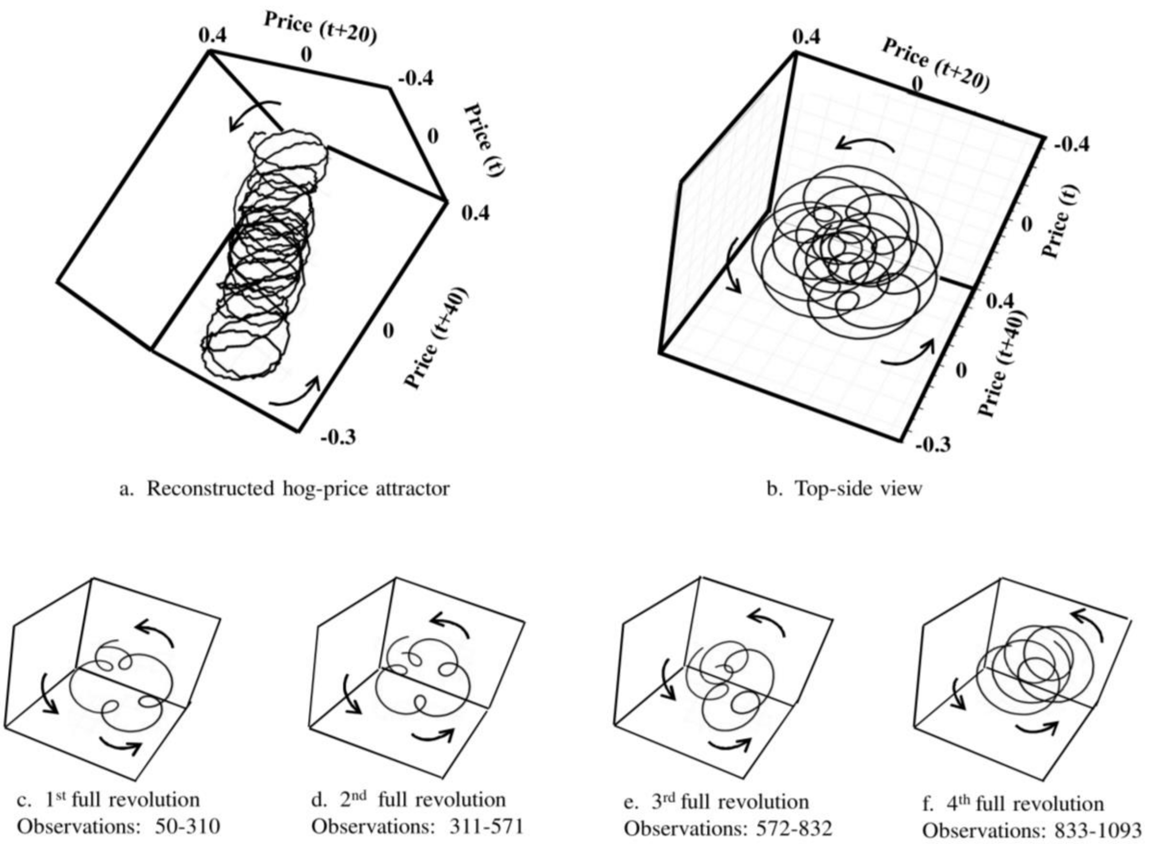


Figure 6. Anatomy of shadow hog-price attractor

5 A nonlinear dynamic model of the German hog industry

The information revealed by nonlinear time series analysis guides our modeling of the German hog industry. The empirical hog-price attractor has an embedding dimension of $m = 3$, indicating that at least three state variables are required to generate empirically-detected system dynamics. The computed Lyapunov exponent supports the hypothesis of divergent, possibly chaotic behavior. If the system is represented in continuous[§§] time, at least three differential equations are necessary to generate chaotic behavior. The empirical attractor is composed of two major cycles. The 5-year cycle could represent an investment pattern, and the annual cycle the short term adjustment of production. Both are linked to the price of slaughter hogs.

We emphasize that our nonlinear hog market model does not require the random supply and demand shifters conventionally used to create volatility exogenously in linear market formulations. Rather, we employ nonlinear feedbacks among system variables to generate systematic volatility endogenously, after having applied signal processing to purge observed hog prices of volatility due to white noise. Moreover, we employ the smallest set of state variables required to reconstruct empirically-detected nonlinear market dynamics, and thus do not attempt to formulate a detailed simulation model.

We propose the following fifth order system of differential equations:

[§§] The system will be modeled in continuous time since all actors are assumed to make their decisions independently at arbitrary points in time. This leads to a continuous time representation of the aggregated flows incorporated in the model. Contrary, a discrete time model would imply that all actions are synchronized as to take place at the discrete time steps of the model which, in our case, would be an unrealistic assumption.

$$\begin{aligned} \dot{P} &= \mathrm{f}_p(D,S,P) \\ \dddot{S} &= \mathrm{f}_s(P,S,C) \\ \dot{C} &= \mathrm{f}_c(P,C) \end{aligned} \qquad (1)$$

The system is composed of three dynamic processes: price adjustment, adjustment of the quantity of supply resulting from production decisions, and adjustment of the production capacities through investments. There are three state variables: production capacities (*C*), actual production or supply (*S*), and hog prices (*P*); along with time lags constituting additional (intermediate) states. The first equation describes the price adjustment process in which price change $\dot{P}$ depends on demand (*D*), supply (*S*) and the current price (*P*). The notation $\dddot{S}$ indicates a third order differential equation that determines supply adjustment. This equation, to be specified later, models the production decisions based on the marginal cost function, and likewise considers the delay caused by the time period necessary to complete the production process. Production decisions in the short run are constrained by available production capacities; in the long run these may be expanded through investments. This process is modeled by the third equation, where the rate of change of production capacities $\dot{C}$ depends on product price (*P*) and current resources (*C*). Given the third order plus two first order differential equations, the above equations comprise a fifth order system.

Operationalizing the model requires specification of the above equations. We begin with the price adjustment. Assuming a trial and error process, the rate of price change can be viewed as dependent on the difference between demand and supply, i.e. (*D – S*). This implies that the actors on the market have crude information on actual prices and trade volumes. This information is available for the German hog market from weekly magazines and the internet. The simplest functional form is a linear relationship, i.e. $\dot{P} = a\left(D-S\right), \quad a>0$. Assuming that large surpluses of either demand or supply speed up the adjustment process, a more adequate formulation is:

$$\dot{P} = a\left(D-S\right)^3 \;, \quad a>0 \qquad (2)$$

Alternatively, we may postulate that the *relative* rate of price change equals the right hand side expression of the above formula:

$$\frac{\dot{P}}{P} = a\left(D-S\right)^3 \quad \text{or} \quad \dot{P} = a\left(D-S\right)^3 P \;, \quad a>0 \qquad (3)$$

This constitutes an additional feedback loop in the model. We use equation (3) in the model.

Figure 7 depicts the dependence of the marginal price change $\dot{P}$ on the difference between demand and supply (*D – S*) and the price level *P* respectively, according to equation (3).

Demand (*D*) is modeled with an isoelastic demand function:

$$D = b\,P^{-c} \;, \quad c>0 \qquad (4)$$

where *c* represents the price elasticity of demand and *b* is a scale factor.

The process of supply adjustment is represented by the third order differential equation $\dddot{S}$ in (1). It can be separated into two components representing (a) the production decisions and (b) the time lag that occurs between the decision to start a production process and its completion. The production decision is based on the marginal cost function of the average production unit and the number of production units currently in service:

$$S_p = C\,g\,P^{\,d} \;, \quad g,d>0 \qquad (5)$$

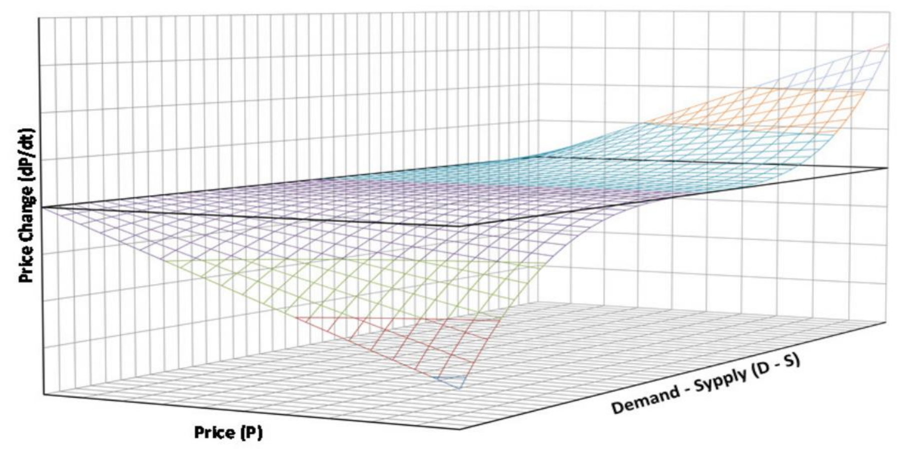


Figure 7. Price change as a function of demand and supply

The process of supply adjustment is represented by the third order differential equation $\dddot{S}$ in (1). It can be separated into two components representing (a) the production decisions and (b) the time lag that occurs between the decision to start a production process and its completion. The production decision is based on the marginal cost function of the average production unit and the number of production units currently in service:

$$S_p = C\,g\,P^d \quad , \quad g, d > 0 \tag{5}$$

S_p represents "planned" supply according to the actual decisions, and $g\,P^d$ represents the marginal cost function. An exponent $d < 1$ indicates economies of scale while $d > 1$ marks diseconomies of scale. If $d = 1$ no scale effects occur.

The production time lag is modeled via an *exponentially distributed delay* which is generally defined by the system of first order differential equations

$$\begin{aligned} &\dot{r}_i = \frac{k}{DEL}\left(r_{i-1} - r_i\right), \quad i = 1,2,\ldots,k \\ &\text{with} \\ &r_0 = S_p \\ &r_k = S \end{aligned} \tag{6}$$

where k marks the order of the delay (in our case k=3), and DEL denotes the average delay time (the production period plus the reaction time of the decision makers).

The adjustment of production capacities follows the differential equation

$$\dot{C} = w\left(1 - \frac{C}{v\,P}\right)C - \frac{C}{l}\,, \qquad w,v,l > 0 \tag{7}$$

The first term represents investments and the second measures the reduction of production facilities due to wear and tear. The parameter l measures the service life of the production facilities. The investment term assumes that the adjustment of production capacities follows a logistic growth process for the case of constant product price P. The term $v\,P$ marks the upper limit of this process, and can be interpreted as a "target size" of the sector proportional to P. A falling market price P can cause disinvestments if the term inside the brackets becomes negative as current capacities C exceed $v\,P$. This negates sunk cost effects that are important in the German hog sector due to the high specificity of the facilities. To allow for irreversibility due to sunk costs, the following formulation was used in the model:

$$\dot{C} = Max\left[w\left(1-\frac{C}{v\,P}\right)C, 0\right] - \frac{C}{l}, \qquad w,v,l > 0 \tag{8}$$

where the *Max*[·] operator ensures that investments are always positive or zero, and capacities can decline only through deterioration.

Large investments often cause high financial leverage that inhibit investments for a period of financial consolidation. This can be factored into equation (8) by introducing a (discrete) time lag:

$$\dot{C} = Max\left[w\left(1-\frac{C(t-T)}{v\,P}\right)C, 0\right] - \frac{C}{l}, \qquad w,v,l > 0 \tag{9}$$

The expression *C*(*t*–*T*) represents production capacities lagged by *T* time units. This formulation is equivalent to the introduction of a maturation delay in logistic population models and may cause a periodicity if the time lag is significant.

Figure 8 summarizes our nonlinear model of the German hog industry.

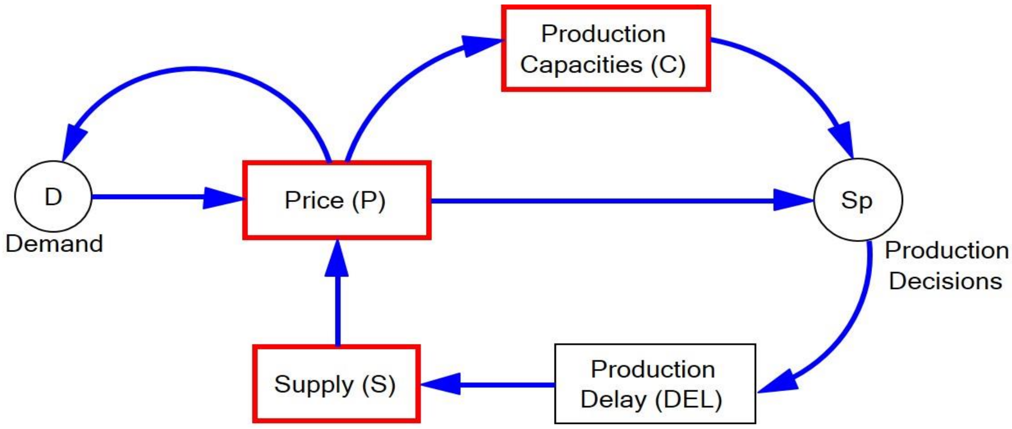


Figure 8. Nonlinear dynamic model of German hog industry

6 Model results

The model was implemented in © *Vensim* and solved using a 4^{th} order *Runge-Kutta* integrator. It was simulated over a period of 50 years. Following our empirical results, the base run set the production delay *DEL* to 1 year and the time lag *T* for financial consolidation after large investments to 5 years. The service life of the facilities (*l*) was assumed to be 15 years on average. No scale effects were considered (i.e. *d*=1). The demand elasticity was set to 0.25. Other parameters were normalized to generate a hypothetical equilibrium price of roughly 1.4 €/kg.

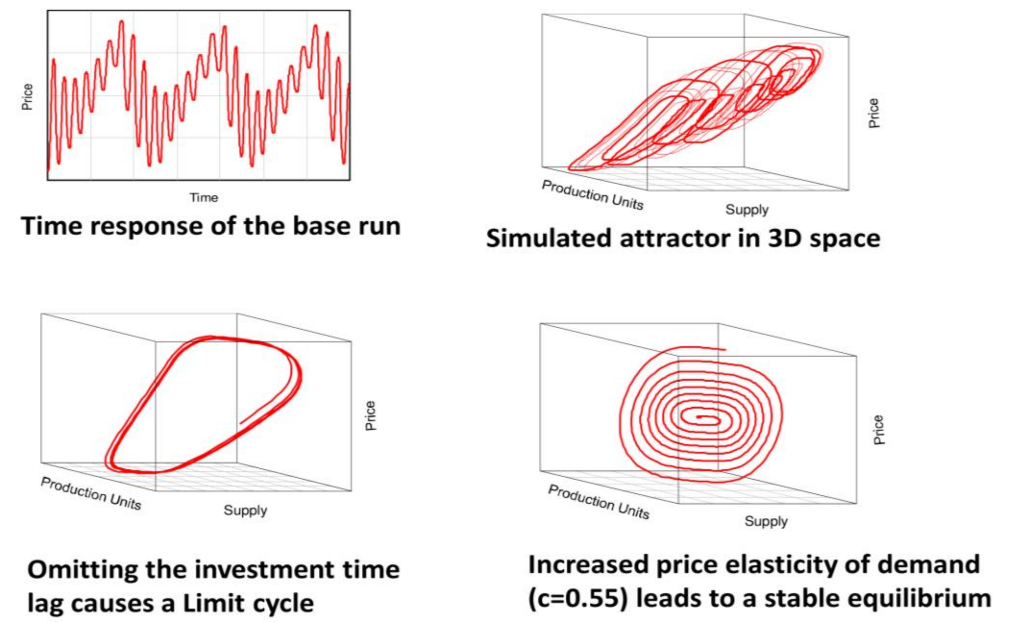


Figure 9. Simulation results

The simulation results are depicted in Figure 9. The price series generated by the base run of the model (Figure 9a) exhibits aperiodic cyclical behavior consistent with the observed hog-price record. Figure 9b portrays the trajectory of the primary state variables of the model, i.e. price, supply and production, in three-dimensional space and thus illustrates the attractor of the system. The graph reveals noticeable similarities with the reconstructed attractor depicted in Figure 6. Reconstructing phase space from the simulated price series results in an embedding dimension of *m*=3 and a time lag of *d*=20, and thus reveals largely the same results as obtained in the reconstruction for the original time series. This indicates that our model matches the dynamic behavior diagnosed for the real world system, and therefore provides a means to identify important determinants for the persistent hog cycle.

Since the model is completely deterministic, the revealed market instability is endogenous and the aperiodic cycling emerges without external shocks. The dynamic properties of the system are due to the inherent nonlinearities along with the built in time lags. The nonlinearities refer primarily to (1) the price adjustment process, (2) the irreversibility of investments due to sunk cost and (3) the logistic type adjustment of production capacities. Together with the periodicity of investments induced by the financial consolidation time lag, these factors result in the dynamic response displayed in the upper part of Figure 9.

With appropriate parameter changes, the model can generate quite different types of dynamic behavior as seen from the trajectories depicted in the lower part of Figure 9. If the financial consolidation time lag is omitted, the simulated attractor is converted into a 'limit cycle'. Regardless of the starting point, all trajectories converge on one orbit (Figure 9c). This behavior is caused by the combination of low demand elasticity and the irreversibility of investments. It holds over a fairly wide range of parameters. Only increased price elasticity of demand changes system dynamics to a 'point' attractor (Figure 9d). In the absence of external shocks, the system approaches a stable equilibrium. However, this is unrealistic because low demand elasticity for food is characteristic for all industrialized countries where only a small portion of income is spent for food.

7 Conclusions

We applied a diagnostic modeling approach to investigate causal factors driving the persistent German hog-price cycle. Nonlinear time series analysis reconstructed an empirical hog-price attractor governing the aperiodic cycling of hog prices over time. Our empirical results indicate that causal factors driving the hog-price cycle are endogenous to the industry, and therefore can be investigated informatively by formulating a structural industry model. We drew from empirically diagnosed industry dynamics, and knowledge of industry structure and technology, to formulate a model that successfully simulated the dynamic complexity of the real-world hog-price cycle.

The model provided important insights into the origin of the hog cycle in Germany. Besides the low price elasticity of demand, which is a well-known determinant of market cycles, the model revealed two more important influence factors. One is the irreversibility of investments caused by the high specificity of the technology. Along with low demand elasticity, this leads to permanent fluctuations in form of a limit cycle. Another important factor is periodicity of investments induced by a time lag forcing a period of financial consolidation after a big investment. This is consistent with the investment behavior of German farmers which is often liquidity driven. It also reflects restrictions on the debt ratio imposed by the capital market. Adding this factor to the model converted the limit cycle into a torus-like attractor.

These results have several practical implications. First, valid medium and longer term price forecasts (i.e. beyond a few weeks) are precluded by the nature of the attractor. By the same token, policy measures aimed at price stabilization (i.e. buffer stock policies) are likely to fail. Accepting that in industrialized countries demand elasticity can hardly be influenced, the remaining starting points for altering the system behavior are (1) the technology and (2) the investment and financing behavior. First, a more flexible technology (e.g. multi-purpose instead of highly specialized facilities) involving less sunk cost would enable a flexible response to changing market conditions, thus lessening the degree of irreversibility of investments. Regarding the second aspect, utilizing alternative ways of financing which focus on equity capital (provided by external investors) rather than bank loans, would help smoothing the investment cycles.

The methodology presented in this paper goes beyond conventional time series modeling – including state of the art methods of price volatility analysis (e.g. GARCH-approaches) – as it not only aims at reconstructing the time pattern of the series, but seeks to identify causal factors driving the system dynamics. To this end, a structural model serves as analytical tool, the design and development of which is guided by the empirically-diagnosed dynamic properties of the system (i.e. the nature of the attractor) along with existing knowledge about the industry. The diagnostic part of the approach is primarily based on *Phase Space Reconstruction* techniques. However, these techniques fail to reveal a clear picture if the investigated time series contains notable (colored) noise, as is the case for most economic time series. *Singular Spectrum Analysis* was therefore applied first, and turned out to be a useful method for constructing a noise-free series for the further analysis that still incorporates the essential system dynamics.

The presented diagnostic modeling approach is applicable to a wide range of problems focusing on the analysis of systems driven by nonlinear dynamics. These systems are often characterized by chaotic attractors whose essential properties can be empirically diagnosed as described, and applied to formulate theory-based models able to simulate the complexity of real-world dynamics.

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