Available online at www.centmapress.org

INTERNATIONAL JOURNAL ON FOOD SYSTEM DYNAMICS

Int. J. Food System Dynamics 11 (1), 2020, 39-51

DOI: http://dx.doi.org/10.18461/ijfsd.v11i1.38

Pricing Perishables: Robust Price Assurance

Robert D. Weaver[1,2] and Yongma Moon[2]

[1,2]Department of Agricultural Economics, Sociology, and Education, The Pennsylvania State University, University Park, PA 16802
[2]College of Business Administration, University of Seoul, Korea
r2w@psu.edu

Received July 2019, accepted December 2019, available online February 2020

ABSTRACT

As perishable products are worthless at end-of-life, for a given supply prices are often dynamically adjusted to ensure inventory is exhausted at end-of-life. When consumers expect such price reductions, they may strategically time their purchases. These two conditions pose a complex problem for pricing. Given inventory, cost of production is sunk. Thus, the dynamic path for prices must be set to maximize revenues with an eye on inventory take-down as well as to discourage strategic behavior. This problem is further challenged when prices and the extent of consumer strategic behavior are uncertain. This paper presents an approach for pricing a set of perishable products that are highly substitutable, yet differentiated to target a set of consumer segments. We propose and analyze a price assurance scheme as a solution to the strategic behavior of consumers and price uncertainty. We present and evaluate our price assurance approach by comparing two price assurance schemes: i) *ex-post* price assurance, and ii) *ex-ante* price assurance to risk neutral dynamic pricing without regard for consumer strategic behavior. These approaches have not to our knowledge been previously considered in our setting of perishables, uncertain consumer strategic behavior, and price uncertainty. Our numerical experiments show that our robust optimization model prevents loss when a firm encounters the worst-case demand and outperforms a risk neutral pricing model. Comparison across our different pricing schemes provides conditions under which particular schemes may dominate others.

Keywords*: Dynamic pricing, price assurance, perishables, product differentiation, strategic consumers, revenue maximization, robust optimization.*

1 Introduction

Management of the economic performance implications of product perishability is a critical component of firm-level profitability. Product perishability may follow from physical attribute dynamics, attribute functional obsolescence, or evolution of buyer preferences. Food sector examples are well-known where product life is short, stochastic in duration, and often requires process or intervention to manage product life, e.g. cool misting fresh vegetables. Digital products are often viewed as having high degrees of attribute obsolescence. Consumer preferences clearly evolve as can be seen in food attribute preferences. The term "fashionable" highlights preference evolution that often leads to viral shifts in demand or more gradual preference diminution. In many cases, such shifts can be abrupt as consumers switch purchases to products characterized by new bundles of attributes. Services provide examples where service-life abruptly goes to zero, e.g. when a flight departs or a hotel room day is closed as vacant at day's end. Information technology has enabled rapid assessment of consumer demand, just-in-time production, and rapid coordination/control technologies, enabling a dramatic expansion of the supply of highly differentiated and substitutable products and services that can be targeted at increasingly granular consumer segments with rapidly evolving preferences. However, those same IT developments have facilitated strategic behavior by customers in response to anticipated dynamics of product availability, attributes, as well as price. That is, anticipating price dynamics, consumers may act strategically to time their purchases, e.g. postponing purchase to wait for anticipated price reductions.

From the firm's perspective, this environment implies consumer demand has become characterized by four important features: 1) uncertainty such that the underlying stochastic process is not obvious and often involves discrete jumps and dynamic, 2) increasingly granular across a wide spectrum of differentiated products serving different market segments, 3) discontinuity between demand and price as consumers switch purchases across products, and 4) strategic behavior by consumers who postpone purchase in anticipation of price reductions.

We adopt and extend the Weaver and Moon dynamic pricing model to address these features. Following Weaver & Moon, we consider dynamic pricing model based on robust optimization to derive dynamic pricing that is consistent with the presence of highly differentiated substitutes, uncertainty of demand, heterogeneity across consumer segments, and strategic behavior by consumers. We consider as a base case where the firm is risk neutral and forms price policy using expectations or point forecasts of demand. We label this pricing scheme as *risk neutral pricing* as it derives optimal price dynamics are risk neutral on point forecasts consistent with risk neutrality. This conception supposes the firm has knowledge of the underlying stochastic distribution of demand. While Weaver and Moon presented price policy that is responsive to the first three features of demand noted above, they did not present price policy that responds to strategic behavior of consumers. We extend their model to consider two forms of price assurance as approaches to mitigate the revenue impacts of strategic behavior by consumers. We consider two types of price assurance policy. We consider an *ex-post* price assurance policy that promises a refund of price reduction to customers who have paid a price that has been subsequently reduced. We consider a second policy that we label as *ex-ante* price assurance in which optimal dynamic prices are derived to explicitly avoid refund liability. While the ex-post price assurance model has been considered by Lai et al. 2009, to our knowledge we present the first consideration of ex-ante price assurance and a direct comparison of three different pricing strategies (risk neutral, ex-post, and ex-ante). Our approach also extends the literature on price assurance by incorporation of highly differentiated, substitutable products that are also perishable as well as by consideration of true uncertainty with a robust optimization approach. Our model provides a discrete-time, dynamic price profile within a context where the firm cannot replenish inventory. Section 2 provides a brief review of literature regarding perishable product pricing, customer's behavior and price assurance, as well as robust optimization. Section 3 presents our risk neutral pricing model as well as variations consistent with two price assurance policies (ex-post and ex-ante). Section 4 explores the implications of our proposed price assurance policies using numerical experiments. Section 5 presents discussion and conclusions.

2 Past Work

As reviewed by Weaver and Moon (2018), an extensive literature exists that considers the revenue management and dynamic pricing problem for perishable products. The Weaver and Moon (2018) model for dynamic pricing incorporated several key features noteworthy: multiple, substitutable products that are perishable, uncertain demand by a heterogeneous, segmented population of consumers, and strategic behavior by consumers who time purchases to exploit anticipated mark-downs. The relevance of multiple, substitutable products introduces the potential for pricing of one product to cannibalize other products. That is, consider management of a vast set of products that are close substitutes such that consumers can switch within the set depending on price and preferences. A change in a particular product price will impact the demand and revenue for other products through smooth substitution or abrupt switching. This implies that pricing across the set of products must be accomplished through joint consideration of the demand interdependence. Consumer heterogeneity implies management can position products and their pricing to segment the market, allowing for an approximation of price discrimination effects. Finally, the strategic behavior of consumers across the products is influenced by budget constraints as noted by Bitran et al. 2006. Zhang and Cooper 2005 and 2009 considered dynamic pricing of the multiple substitutable flights and provided bounds to the optimal revenue using a heuristics solution approach. Digital media enables customers to observe price dynamics and are enabled to plan the timing of purchase. Su 2007 developed an intertemporal pricing model which considers heterogeneous customer preferences as well as different degrees of patience. Aviv and Pazgal 2008 also consider patience. Firm-level pricing with consumer price taking implies the firm can be viewed as a leader in a Stackelberg game. Aviv and Pazgal 2008 consider performance of a pre-announced fixed discount policy. Both Aviv and Pazgal 2008 and Levin et al. 2009 show that customer strategic behavior generally will reduce firm revenues.

An alternative response to consumer strategic behavior is to adopt a price assurance policy. Such a policy dissolves incentives for consumers to postpone purchase. Levin et al. 2007 presented a price assurance model in a monopolistic setting where customers are reimbursed for the price difference between a purchase price and a reduced price that occurs after purchase. Customers are partitioned into three segments: 1) do not buy, 2) buy without and 3) buy with a price guarantee option. Levin et al. show that faced with uncertain dynamic pricing, a price assurance option has option value. Lai, et al. 2009 further studied such an *ex-post* price assurance policy specifying consumers as strategic with total demand uncertainty. That is, price adjustment occurs *ex-post* a price change. Under their *ex-post* price assurance specification price is derived in a risk neutral setting and refunds are paid after a price reduction occurs. They found that the policy eliminates strategic consumer postponement.

To proceed, we note the need for consideration of uncertainty in our problem setting. Much of past literature has not considered risk or uncertainty. Risk neutral settings have received limited consideration while uncertainty noted by Weaver and Moon 2018 more accurately characterizes the setting. Bertsimas and Sim 2004 noted the sensitivity of dynamic pricing to stochastic specifications. In particular, risk neutral specifications ignore the implications for performance of worst case scenarios and are difficult to implement when characteristics of the underlying stochastic distribution are unknown. The idea behind *robust optimization* is to consider the worst case scenario without a specific distribution assumption. Ben-Tal and Nemirovski 1999, Bertsimas and Sim 2003, and Ben-Tal et al. 2004 addressed models that define different uncertainty sets such as an ellipsoidal, polyhedral and cardinality set. The robust optimization method has been widely used in many different applications such as finance and transportation area to respond to uncertainty while preventing a certain degree of loss. We adopt the Weaver and Moon (2008) specification as briefly presented below and extend it to consider price assurance. Our price assurance policies are illustrated using numerical experiments and show that performance under price assurance dominates previous dynamic pricing presented by Weaver and Moon (2018).

3 Model Overview

In this section, we summarize our approach as based on several key components. First, we suppose the producer has produced a set of differentiated, perishable products. A good example might be cut Christmas trees of different varies and heights, harvested fruit with different characteristics, or cell phones. Cost of production is already paid, so it is a sunk cost. We suppose consumers are different, heterogeneous, and can be thought of as composing a set of stratified, different consumer segments, each with different demand functions ranging from one with the highest willingness-to-pay to the lowest. We assume demand for each product is responsive to its own price as well as the price of the other products. We suppose demand goes to zero at the end of product life, or market season. We assume the producer has estimates of each segment's demand functions and so can predict, though with error, demand given a set of prices. Thus, the producer cases uncertainty with respect to segment specific demand. We assume the producer and consumer knows the product life and the need for the producer to sell available inventory. Thus, we suppose that some consumers may act strategically by postponing purchase in hope of paying a lower price. This is a very real setting faced for most products with a product life.

For the producer, the challenge is to set a dynamic path for prices across the set of products to maximize revenue given the setting faced in the market. Price could be set as fixed throughout the product life, or an ad hoc adjustment might be done through a sale at some point in time. However, we assume the producer will choose a dynamic path for prices that reflects the market setting and optimizes revenue performance. Given demand is stochastic, for a base case, we suppose the producer is risk neutral and chooses an optimal price path to maximize expected revenue conditional on the producer's estimates of strategic behavior by consumers. Next, we generalize the problem to suppose the producer faces true uncertainty with respect to demand, i.e. the producer can only describe the boundaries (worse and best cases) for demand. In this setting, we suppose the producer sets dynamic price paths through robust optimization. Further, within this setting we evaluate two pricing policies we call ex-post and ex-ante price assurance. That is, for ex-post price assurance, we suppose the producer seeks to mitigate strategic postponement by consumers by offering to refund the price difference between the purchase price paid and lowest price available during the marketing season. Next, we consider an alternative price assurance policy where the producer derives an robust dynamic price path such that consumers can be guaranteed that prices are not reduced within the market season. This, of course, is enforced by a constraint on the producer's robust optimization problem.

To proceed, we briefly summarize notation for our model and refer the reader to the appendix for details. Our approach relies on notation introduced by Weaver and Moon 2018.

Table 1.
Notation

J	# of products offered by producer, $j=1,...,J$
t	time date within market season, $t=0,1,...T$
T	End date of market season, product life length
s	Consumer segment, $s \in S_0 = \{1,2,...,S\}$
$q_s^j(0)$	Initial product inventory available to segment *s*
$q_s^j(t)$	Product inventory at time *t*
$p_s^j(t)$	Price of product *j* set for demand segment *s* at time *t*
$\delta_j^s(t)$	Demand by segment *s* at time *t* for product *j*
$\eta_s^j(t)$	Proportion of consumers in demand segment *s* at time *t* who postpone purchase of product *j*
$a_s^j(t)$	market potential for product *j* and segment *s* at time *t*
$\beta_s^j(t)$	price sensitivity of product *j*
$\gamma_s^j(t)$	price sensitivity of substitutes for product *j*

Demand for substitutable, perishable products

We specify demand functions as follows:

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t)p_s^k(t) \qquad \forall s, j, t$$

We assume that each segment prefers finding substitutable products rather than downgrading or upgrading to a different segment's product set. For example, price sensitive customers want to purchase an airline ticket among substitutable business class flights rather than to upgrade from an economy class ticket.

Demand postponement by Strategic Consumers

Building on Su and Zhang 2008 and Lai, et al. 2009, we assume strategic behavior by customers occurs through postponement of purchase. The result is that demand at time t is pushed into the next time period $t+1$. We define the cumulative demand function as $\tilde{\delta}_s^j(t)$ as:

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))) = (1-\eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))],\ t = 1...T,\ \forall s, j$$

That is, a proportion of current demand $\delta_s^j(p_s^j(t), p_s^{-j}(t))$ is postponed, and a proportion of demand from the previous time $\eta_s^j(t-1)\tilde{\delta}_s^j(t-1)$ is brought to the current time. This leads to total demand at time *t* as the square-bracketed portion which in turn is subject to further postponement by some consumers.

Dynamic Risk Neutral Pricing of Perishables

We assume the firm's management problem involves setting a dynamic price path to maximize expected revenue from each consumer segment based on several constraints: 1) initial and current inventory, 2) current total demand given past and current postponement, and 3) demand, price, and inventory must be non-negative. Expected revenue is defined at the summation over products, segments, and time of expected price times expected quantity purchased.

Weaver-Moon Pricing under uncertainty with robust optimization

In contrast to the case of risk neutral dynamic pricing, to consider price assurance we now consider a more realistic setting where we suppose the firm has limited knowledge of the stochastic properties of product and segment specific demand. Faced with such uncertainty, we characterize the optimal price profiles as following from a robust optimization following i.e. Lan, et al. 2008 Birbil, et al. 2009, Weaver and Moon. 2018. Our proposed robust optimization model follows Soyster 1973 and Ben-Tal and Nemirovski 1999. At the core of this approach, we suppose the values of stochastic variables such as potential demand fall with a set values defined by a range of deviation around a mean value, so e.g. for potential demand:

$$a_s^j(t) \in [\bar{a}_s^j(t)(1-\theta_s^j(t)), \bar{a}_s^j(t)(1+\theta_s^j(t))]$$

This uncertainty set can be defined to be consistent with a particular setting. For example, it can be made less conservative by assuming many different types of uncertainty set or robust counterparts such as ellipsoidal and polyhedral uncertainty sets as considered by Ben-Tal and Nemirovski 1999 and Bertsimas and Sim 2004. Given the uncertainty set, the optimal price policy should satisfy the revenue optimization problem presented above. To proceed, it is practice to translate the problem into tractable form as reported in the Appendix. Weaver and Moon 2018 showed that this robust optimization model sets prices lower than a risk neutral model. This implies that a firm attempts to expand sales and inventory to draw down inventory by setting prices lower under uncertain demand. Their results also illustrated segment-specific pricing that reflects a preference ordering across consumer segments that sets boundary thresholds beyond which pricing can induce consumers to switch products. They also illustrate that robust pricing dominates risk neutral pricing across a range of performance criteria. Compared to risk neutral pricing, robust pricing is shown to result in more stable prices as well as revenue over time.

Dynamic Pricing with Price Assurance

We extend the Weaver and Moon 2018 robust optimization model to incorporate price assurance policy. We consider both *ex-post* and *ex-ante* price assurance policy based on respective robust pricing models and then compare the performance implications of the policies. Given the two price policies, we are able to evaluate the following proposition.

Proposition 1. Given demand uncertainty, the proportion of strategic customer postponement $\eta_s^j(t)$ is reduced by a policy of robust price assurance relative to that under a risk neutral pricing policy.

While Lai, et al. 2009 found price assurance policy eliminates consumer postponement under risk neutral pricing, our extended proposition considers the implication of price assurance within a context of uncertainty.

Under ex-post price assurance, we specify that the firm offers to refund the price difference between the purchase price at time t ($p_s^j(t)$) and lowest price available in the marketing season. We suppose the refund is claimed by a proportion of buyers $\omega_s(t)$< 1. This policy comes at a direct cost that is simply the proportion of claims made times total purchases times the difference between price paid and lowest price in the season at some time τ, i.e.

$$\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=0}^{T-1}(p_s^j(t)-\min_{\forall\tau>t}p_s^j(\tau))^+\omega_s(t)\tilde{\delta}_s^j(t) \text{ where } X^+=\max(X,0).$$

This cost must be incorporated as a reduction in revenue to define the robust price assurance model as reported in the Appendix.

Ex-ante price assurance

As an alternative to the ex-post price assurance policy, we consider price policy that dynamically prices to eliminate refund liability by applying the intuitive condition that prices are not reduced within the market season, i.e. the following constraint is set on the optimal price path as shown in the Appendix:

$$p_s^j(t)\geq p_s^j(t-1) \text{ for } t=1,...,T.$$

4 Performance Implications of Price Assurance

Here, we consider the performance implications of three dynamic pricing methods: i) risk neutral without considering price assurance, ii) robust ex-post price assurance and iii) robust ex-ante price assurance. These problems are not analytic and so, we illustrate their implications and establish their tractability using numerical methods within the context of a set of numerical experiments. Our numerical experiments are based on parameterization in Table 2 used by Weaver and Moon 2018. As in that paper, our goal is to illustrate how the approach can be used by illustrating it within the context plausible parameter values. For further applications, estimates of these parameters could be used based on the specific applied setting of interest. To summarize, estimates would be needed for 4 parameters for each consumer segment to describe the demand curve (intercept, $a_s^j(t)$, price response $\beta_s^j(t)$ and $\gamma_s^j(t)$, and proportion of strategic consumers $\eta_s^j(t)$), one parameter describing the bounds of the uncertainty set, $\theta_s^j(t)$, and a parameter describing the proportion of consumers that would claim a refund under ex-poste price assurance, $\omega_s(t)$. Thus, in a particular applied setting such relatively little information (in the form of parameter estimates) is needed.

Table 2.
Parameters for $j=2$, $s=2$, $T=10$

Parameters	Product 1		Product 2	
	Segment 1	Segment 2	Segment 1	Segment 2
$a_s^j(t)$	60	120	30	100
$\beta_s^j(t)$	0.5	1.5	0.6	1.6
$\gamma_s^j(t)$	0.1	0.3	0.1	0.3
$\eta_s^j(t)$	0.2	0.2	0.2	0.2
$\omega_s(t)$	0.5	0.5	0.5	0.5
$\theta_s^j(t)$	0.02	0.02	0.02	0.02
$q^j(0)$	100	400	80	300

The experiments are implemented using MATLAB and GAMS on a machine with Windows XP OS, T2300 CPU, 1.66 GHz, and 1 GB RAM.

Implications of uncertainty and customer behavior

As noted by Bertsimas and Sim 2004, the specification of stochastic forces in the problem faced by the firm will clearly affect performance of the firm's choices. In our risk neutral price policy, we assume the firm knows the underlying distribution of demand stochastics and uses expected demand functions in its derivation of optimal price. In our price assurance cases, we suppose the firm faces uncertainty, not risk, and can only characterize the bounds of the uncertainty set of demands it faces. By comparing robust price assurance results with those for the risk neutral price policy, we can evaluate the implications of uncertainty and schemes that attempt to reduce its revenue implications. The performance of each dynamic pricing method depends on the nature of consumer behavior with respect to propensities to postpone and to claim refunds. We define R_{cont}, R_{post}, R_{ante} as revenue generated by a risk neutral, ex-post price assurance, and ex-ante price assurance policies, respectively. We consider revenue performance across our price policy schemes and across a range of values characterizing consumer postponement ($\eta_s^j(t)$) and consumer propensity to claim refunds ($w_s(t)$). The results are reported in Table 3.

Table 3.
Simulation experiments for different customer behaviors

$\eta_s^j(t)$	0		0.1		0.2		0.2
$w_s(t)$	R_{post}	R_{ante}	R_{post}	R_{ante}	R_{post}	R_{ante}	R_{cont}
0.3	5.9556e+004	6.3480e+004	4.8749e+004	8.5010e+003	4.6388e+004	8.1104e+003	4.6854e+004
0.5	5.8568e+004	6.3480e+004	4.8453e+004	8.5010e+003	4.6077e+004	8.1104e+003	4.6854e+004
0.7	5.7059e+004	6.3480e+004	4.8158e+004	8.5010e+003	4.5766e+004	8.1104e+003	4.6854e+004
0.9	5.6251e+004	6.3480e+004	4.7862e+004	8.5010e+003	4.5456e+004	8.1104e+003	4.6854e+004

We first suppose the presence of price assurance does not affect strategic behavior by consumers. That is, we consider the case where the proportion of postponed demand by strategic consumers is the same under each price policy. For $\eta_s^j(t)$=0.2, comparing the last two columns of Table 2, smaller revenues are achieved under ex-post and ex-ante price assurance relative a risk neutral pricing policy. Compared to risk neutral pricing revenue R_{cont}, the revenue reduction for ex-post price assurance is small, ranging from -0.99% to -0.029% for propensity to claim refunds $w_s(t)$ from 0.3 to 0.9. Compared to risk neutral pricing revenue, revenue reduction for ex-ante price assurance is substantial at -82.7%. These results suggest that ex-post price assurance can be costly depending on the consumer propensity to claim refunds, however the revenue reduction for ex-ante price assurance is shown to be substantial. Based on results reported by Weaver and Moon 2018 for revenue variation with respect to the extent of uncertainty, θ, the reductions in revenue could be greater. This cost of revenue reduction is incurred directly in refunds under ex-post assurance and as a result of constraints under ex-ante assurance. As propensity to claim refunds increases under ex-post assurance, Table 3 shows the revenue difference is slightly reduced between ex-post versus ex-ante price assurance.

We now consider the case where price assurance policy reduces the extent of strategic consumer behavior. Proposition 1 supposes that price assurance would lead consumers to reduce their strategic postponement. If the offer of price assurance eliminates strategic behavior by consumers ($\eta_s^j(t)$=0), Table 3 shows that both price assurance policies provide increased revenue relative to the risk neutral model. Comparing across columns in Table 3, when postponement exists, we see that ex-post and ex-ante revenues decrease as the propensity to postpone purchases increases. Table 3 also provides a comparison of revenue performance under price assurance for $\eta_s^j(t)$=0 versus risk neutral price policy without price assurance for $\eta_s^j(t)$=0.2. In other words, this comparison supposes postponement is eliminated by offering price assurance. Results in Table 3 indicate that if strategic behavior were eliminated, then under ex-ante price assurance revenue would increase relative to both risk neutral dynamic pricing and ex-post assurance. Relative to risk neutral dynamic pricing, ex-ante price assurance would increase revenue by 35.5% while ex-post would increase revenue by 27.1%. As propensity to claim refunds increases, ex-post price assurance revenue decreases. Comparing the price assurance policies, the ex-ante model outperforms the ex-post model for $\eta_s^j(t)$=0. If price assurance policy only reduces strategic behavior, results for $\eta_s^j(t)$=0.1 show that only ex-post assurance would result in increased revenues.

Time-variant demand

Numerical experiments so far have been reported for time-invariant demands. However, in the real world, a firm anticipates increasing or decreasing demand over time. Thus, in this subsection, we investigate how revenues change for increasing or decreasing demand conditions. First, we consider the case of decreasing demand, in other words,

$$a_s^j(t) > a_s^j(t+1), \ \forall t = 1,\dots,T$$

Then, we examine which policy would be more beneficial when demand decreases. For numerical experiments, we assume parameters as follows and keep other parameters the same as in Table 2:

$a_1^1(t) = 60 - kt$, $a_2^1(t) = 120 - kt$, $a_1^2(t) = 30 - kt$, and $a_2^2(t) = 100 - kt$, for $k \in \Re_{++}$.

$\eta_s^j(t)$=0.2 for a risk neutral model,

$\eta_s^j(t)$=0 for an ex-ante and an ex-post model

$w_s(t)$=0.7 for an ex-ante model

The results for decreasing demands are described in Table 4 and the left panel (a) in Figure 1.

Table 4.

Simulation experiments for decreasing demands (values in parenthesis represents rate of decrement, $w_s(t)$=0.7)

k	R_{con}	R_{post}	R_{ante}
0	4.6854e+004	5.7059e+004	6.3480e+004
1	4.1434e+004 (-11.6%)	4.7689e+004 (-16.2%)	5.4607e+004 (-14.0%)
2	3.5318e+004 (-14.8%)	3.2438e+004 (-32.0%)	5.0602e+004 (-7.3%)
3	1.6396e+004 (-53.6%)	6.2442e+003 (-80.8%)	3.0674e+004 (-39.4%)

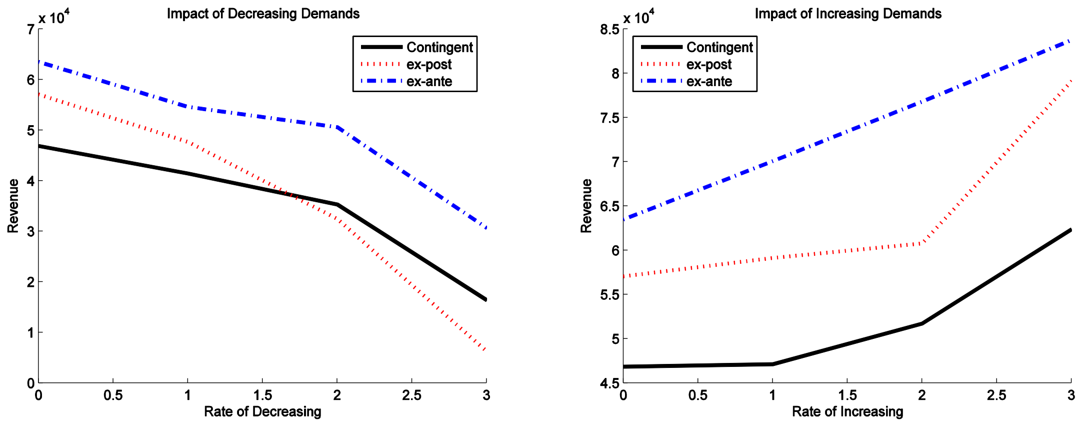

Figure 1. The impact of time-varying demand on different pricing models (a) decreasing demand (left), (b) increasing demand (right), $w_s(t)$=0.7, risk neutral case is indicated as contingent.

Using similar scenarios, we investigate the impact of increasing total demand over time and compare the three pricing schemes. To express increasing demand over time, we assume parameters as follows:

$a_1^1(t) = 60 + kt$, $a_2^1(t) = 120 + kt$, $a_1^2(t) = 30 + kt$, and $a_2^2(t) = 100 + kt$ for $k \in \Re_{++}$.

The results are presented in Table 5 and Figure 1b. Under each growth rate scenario, ex-post price assurance dominates risk neutral pricing and ex-ante price dominates ex-post price assurance with respect to revenue.

As with experiments with respect to strategic behavior and propensity to claim refunds, these results show dramatically that the revenue implications of price assurance schemes strongly depend on market conditions and consumer behavioral response. In a mature market where demand is not expanding, price assurance policy may reduce revenues. Faced with declining demand with high propensity to claim refunds, ex ante price assurance may be very beneficial. Similar to the case of decreasing demands, the ex-ante model has the smallest rate of change in revenue performance as demand growth is increased. Ex-post price assurance and risk neutral pricing is most nonlinear in their rates of change in revenue.

Table 5.

Simulation experiments for increasing demands (values in parenthesis represent rate of increase, $w_s(t)$=0.7)

k	R_{con}	R_{post}	R_{ante}
0	4.6854e+004	5.7059e+004	6.3480e+004
1	4.7128e+004 (0.6%)	5.9136e+004 (3.6%)	7.0074e+004 (10.4%)
2	5.1732e+004 (9.8%)	6.0781e+004 (2.8%)	7.6802e+004 (9.6%)
3	6.2379e+004 (20.6%)	7.9116e+004 (30.2%)	8.3775e+004 (9.1%)

5 Conclusions and further Studies

In this paper, we presented and evaluate three methods for dynamic pricing of perishables: i) risk neutral ii) ex-post price assurance, and iii) ex-ante price assurance. By applying a robust optimization method, we presented robust dynamic pricing policies under demand uncertainty which guarantee a certain level of performance for possible scenarios, even for the worst-case scenario. In other words, our robust models can prevent a realization of a negative demand shock from impacting revenue significantly. Then, we compared different pricing schemes incorporating price assurance schemes. The revenue performance implications of consumer strategic behavior and propensity to claim refunds under various market demand dynamic conditions. Our numerical experiments show that robust price policy tends to stabilize revenues. Results indicate that the revenue implications of negative demand shocks are mitigated by robust policy and can result in greater revenues that under risk neutral price policy. Our results show that price assurance policy outperforms a risk neutral pricing model, when it can eliminate or significantly reduce the customer's strategic postponement of purchases. We find this feature of consumer behavior may be a more important element of revenue management that the propensity to claim refunds. When demand is anticipated to be trending and with respect to revenue enhancement, we find the ex-ante pricing model dominates other dynamic pricing method.

With respect to further research, following the suggestion of Bertsimas and Sim 2004, our results encourage future research to explore how uncertainty conditions may affect revenue performance of dynamic price policy. Our approach and results motivate future applications across perishable products. While many product production processes and supply chains have been re-designed to be just-in-time, many product and service supplies are inherently time-intensive. This implies production and market supply must be based on estimates of uncertain demand. Often, supply realized is also highly stochastic. This market condition deserves further research as an extension of the dynamic pricing problems presented here.

References

Atamturk, A., Zhang, M. (2007). Two-Stage Robust Network Flow and Design Under Demand Uncertainty. *Operations Research*, **55**(4): 662.

Aviv, Y., Pazgal, A. (2008). Optimal Pricing of Seasonal Products in the Presence of Forward-Looking Consumers. *Manufacturing Service Operations Management*, **10**(3): 339-359.

Ben-Tal, A., Goryashko, A., Guslitzer, E., and Nemirovski, A. (2004). Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, **99**(2) 351-376.

Ben-Tal, A., Nemirovski, A. (1999). Robust solutions of uncertain linear programs. *Operations Research Letters*, **25**(1): 1-13.

Ben-Tal, A., Teboulle, M. (2007). An old-new concept of convex risk measures: The Optimized Certainty Equivalent. *Mathematical Finance*, **17**(3): 449-476.

Bertsimas, D., Sim, M. (2003). Robust discrete optimization and network flows. *Mathematical Programming*, **98**(1-3): 49-71.

Bertsimas, D., Sim, M. (2004). The Price of Robustness. *Operations Research*, **52**(1): 35-53.

Birbil, S.I., Frenk, J.B.G.,. Gromicho, J.A.S , and Zhang, S. (2009). The Role of Robust Optimization in Single-Leg Airline Revenue Management. *Management Science*, **55**(1): 148-163.

Bitran, G., Caldentey, R.(2003). An overview of pricing models for revenue management. *Manufacturing & Service Operations Management,* **5**(3): 203-229.

Bitran, G., Caldentey, R., and Vial, R. (2006). Pricing policies for perishable products with demand substitution. City.

Dong, L., Kouvelis, P., and Tian, Z. (2009) Dynamic Pricing and Inventory Control of Substitute Products. *Manufacturing & Service Operations Management*, **11**(2): 317-339.

El Ghaoui, L., Oks, M., and Oustry, F.(2003). Worst-case value-at-risk and robust portfolio optimization: A conic programming approach. *Operations Research*, **51**(4): 543-556.

Elmaghraby, W., Keskinocak, P. (2003). Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science,* **49**(10): 1287-1309.

Gallego, G., Iyengar, G., Phillips, R., and Dubey, A. (2004). Managing flexible products on a network. Department of Industrial Engineering and Operations Research, Columbia University, CORC Technical Report TR-2004-01.

Karoonsoontawong, A., Waller, S.T. (2007). Robust Dynamic Continuous Network Design Problem. *Transportation Research Record*, **2029**(-1): 58-71.

Lai, G., Debo, L.G., and Sycara, K. (2009). Buy Now and Match Later: Impact of Posterior Price Matching on Profit with Strategic Consumers. *Manufacturing Service Operations Management*, doi 10.1287/msom.1080.0248.

Lan, Y., H. Gao, M.O. Ball, I. Karaesmen. (2008). Revenue Management with Limited Demand Information. *Management Science*, **54**(9): 1594-1609.

Levin, Y., McGill, J., and Nediak, M. (2007). Price guarantees in dynamic pricing and revenue management. Operations Research. 55(1) 75-97.

Levin, Y., McGill, J., and Nediak, M. (2009). Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition. *Management Science*, **55**(1): 32-46.

Liu, Q., van Ryzin, G. (2008). On the Choice-Based Linear Programming Model for Network Revenue Management. *Manufacturing Service Operations Management*, **10**(2): 288-310.

Maglaras, C., Meissner, J. (2006). Dynamic pricing strategies for multi-product revenue management problems. *Manufacturing and service operations management*, **8**(2): 136-148.

Perakis, G., Sood, A. (2006). Competitive Multi-period Pricing for Perishable Products: A Robust Optimization Approach. *Mathematical Programming*, **107**(1): 295-335.

Soyster, A.L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*, **21**(5): 1154-1157.

Su, X. (2007). Intertemporal pricing with strategic customer behavior. *Management Science*, **53**(5) 726.

Su, X., Zhang, F. (2008). Strategic Customer Behavior, Commitment, and Supply Chain Performance. *Management Science*, **54**(10): 1759-1773.

Talluri, K., Van Ryzin, G. (2005). The theory and practice of revenue management. Springer Verlag.

Weaver, R.D., Moon, Y. (2018). Pricing Perishables with Uncertain Demand, Substitutes, and Consumer Heterogeneity. *International Journal of Food System Dynamics*, **9** (5): 484-495.

Zhang, D., Cooper, W.L. (2005).Revenue management for parallel flights with customer-choice behavior. *Operations Research*, **53**(3): 415-431.

Zhang, D., Cooper, W.L. (2009). Pricing substitutable flights in airline revenue management. European *Journal of Operational Research*, **197**(3): 848-861.

Appendix: Mathematical Model (see notation in Table 1)

Revenue maximization under risk neutrality

Initial inventories are defined in equation (6), demand is described by equations (2)-(4). Given our specification of ordered segments, to specify products as differentiated products across different segments, we introduce the constraint (8) following Birbil, et al. 2009. This implies that products in $s \in S_0$ have lower prices than higher cost products marketed for segment $s' \in S_0$. Note that the equality holds when a firm does not need to distinguish between the two segments. The revenue optimization problem of the supplier subject to constraints discussed above can be written as follows:

$$R(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t), \forall j,s,t} \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} [p_s^j(t) \tilde{\delta}_s^j(p_s^j(t);t))] \tag{1}$$

s.t. $q_s^j(t) = q_s^j(t-1) - \tilde{\delta}_s^j(p_s^j(t), p_s^j(t))), \quad t = 1...T, \ \forall s, \forall j$ (2)

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))) = (1-\eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))], \ t = 1...T, \ \forall s, j \tag{3}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t), \ \ \forall s, j, t \tag{4}$$

$$q^j(t) \geq 0, \qquad \forall t, j \tag{5}$$

$$q^j(0) = q_0^j, \ \ \forall j \tag{6}$$

$$p_s^j(t) \geq 0, \ \ \forall t, j, s \tag{7}$$

$$p_s^j(t) \geq p_{s'}^j(t), \ \ s > s' \in S_0 \tag{8}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}) \geq 0 \tag{9}$$

$$\tilde{\delta}_s^j(p_s^j(0), p_s^{-j}(0)) = \delta_s^j(p_s^j(0), p_s^{-j}(0)), \ \forall s, j \tag{10}$$

We interpret this problem as a risk neutral pricing approach that is consistent with risk neutrality given that demand parameters can be viewed as expectations of stochastic parameters.

Robust optimization for dynamic pricing

We adopt the following deterministic form that is consistent with risk neutral use of expectations as equivalent to our problem (see appendix A in Weaver and Moon (2018) for derivation):

$$\mathrm{R}(\mathrm{p}_s^j(\mathrm{t})) = \max_{\mathrm{p}_s^j(\mathrm{t}), \forall \mathrm{j,s,t,V}} \mathrm{V} \tag{11}$$

s.t.

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \right\} \right] \geq V \tag{12}$$

$$\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \geq 0 \tag{13}$$

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{i} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau)) - \beta_s^j(\tau) p_s^j(\tau) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(\tau) p_s^k(\tau) \right) \right) \geq 0, \forall s, j, t \tag{14}$$

$$p_s^j(t) \geq 0, \ \forall t, j, s \tag{15}$$

$$p_s^j(t) \geq p_{s'}^j(t), \ s > s' \in S_0 \tag{16}$$

Ex-poste Price Assurance

$$R_{post}(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t), \forall j, s, t, V} V \tag{17}$$

s.t.

$$\begin{aligned}
&\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \right.\right. \\
&\quad \left.\left. + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \right\} \right] \\
&\quad - \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T-1} (p_s^j(t) - \min_{\tau > t} p_s^j(\tau))^+ \omega_s(\tau) \\
&\quad \times \left(\left\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \right.\right. \\
&\quad \left.\left. + + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \right\} \right) \geq V
\end{aligned} \tag{18}$$

$$\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \geq 0 \tag{19}$$

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{i} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau)) - \beta_s^j(\tau) p_s^j(\tau) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(\tau) p_s^k(\tau) \right) \right) \geq 0, \forall s, j, t \tag{20}$$

$$p_s^j(t) \geq 0, \ \forall t, j, s \tag{21}$$

$$p_s^j(t) \geq p_{s'}^j(t), \ s > s' \in S_0 \tag{22}$$

Ex-ante price assurance

Adding this constraint to our robust optimization problem, we derive the ex-ante price assurance policy as follows:

$$R_{ante}(p_s^j(t)) = \max_{p_s^j(t),\forall j,s,t,V} V \tag{23}$$

s.t.

$$\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=0}^{T}\left[p_s^j(t)\left\{\left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0) + \sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\left(\bar{a}_s^j(t)(1-\theta_s^j(t))-\beta_s^j(t)p_s^j(t)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(t)p_s^k(t)\right)\right\}\right]\geq V \tag{24}$$

$$\bar{a}_s^j(t)(1-\theta_s^j(t))-\beta_s^j(t)p_s^j(t)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(t)p_s^k(t)\geq 0 \tag{25}$$

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1}\left(\left(\prod_{\tau=1}^{i}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0) + \sum_{\tau=1}^{i}(1-\eta_s^j(\tau))^{t-\tau}\left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau))-\beta_s^j(\tau)p_s^j(\tau)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(\tau)p_s^k(\tau)\right)\right)\geq 0, \forall s, j, t \tag{26}$$

$$p_s^j(t)\geq 0\,,\ \ \forall t, j, s \tag{27}$$

$$p_s^j(t)\geq p_{s'}^j(t)\,,\ s > s'\in S_0 \tag{28}$$

$$p_s^j(t+1)\geq p_s^j(t) \tag{29}$$